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A SECONDARY ARITHMETIC

COMMERCIAL AND INDUSTRIAL

FOR

HIGH, INDUSTRIAL, COMMERCIAL, NORMAL SCHOOLS,
AND ACADEMIES

BY

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PREFACE

THE educational theory of a people is determined by the dominant social characteristics of the period. The dominant characteristic of the present time in America is that of commercial and industrial growth. Accordingly, we are experiencing a revolution in educational theory quite in keeping with this commercial and industrial development. The greatest movement in education to-day is that for *vocational training*. In fact, education always has been conceived as a preparation for vocation; but our ideas of a vocation and our theory as to how to best prepare for it both are undergoing decided change. President Roosevelt voiced the present tendencies in education in an address to the teachers attending a recent meeting of the Superintendent's section of the N. E. A., when he said: "I trust that more and more our people will see to it that the schools train toward and not away from the farm and workshop."

The courses of study of our high schools have been arranged, formerly, largely with a view to preparing for admission to college; and the college student in turn generally was preparing for some one of the learned professions. That so small a per cent of high school graduates go to college has been a sufficient reason for a radical change in the courses of study of the high schools. The course of study of the progressive high school is not now planned merely to prepare for college entrance, but rather to fit the student for the world's work—to *prepare the great mass of students for a commercial or industrial vocation.*

Commercial and industrial courses are being emphasized. The recent development of separate trade schools, of Y. M. C. A. industrial courses, etc., has been for the purpose of meeting the increasing demand for a practical training for efficiency in industrial vocations.

One of the most essential subjects in a vocational education is a course in commercial and industrial arithmetic. The movement toward a place for this subject in the modern high school course is encouraged by a large number of educators and very generally by the demands of practical men of affairs in the commercial and industrial world. One state and many cities have made arithmetic compulsory in the high schools.

Such a course in arithmetic should be practical and informational. It should be free from puzzles and obsolete processes. *It should set forth the actual business practices of the time.* It should contain nothing but the processes actually used in commercial and industrial work to-day. *And the problems should all be real; that is, they should be just such as are actually met in practical life in commercial work and in the various trades.*

With the present so-called advanced text-books on arithmetic, a course of any great practical value in the secondary school is impossible, except with teachers of broad experience who are familiar with the actual conditions and problems of a mathematical nature encountered in the various trades and in business, and who have been able to extract from these arithmetics the little that is of real practical worth and to supplement it with the real problems that arise in the work of the world about.

To meet the demand of the times for a rational, practical, secondary arithmetic which sets forth the actual business practices of the day, and just these, and which contains a large collection of the actual problems encountered in the various building trades, shop work, agriculture, etc., the authors have

prepared this book. *The book is especially designed to be vocational in its aim.* The part devoted to the commercial applications has been carefully inspected and approved by practical business men. For the material of an industrial nature the authors have gone to men at work in machine shops, in the various building trades, etc. All of the problems are direct, real, and practical.

The book is divided into four parts.

Part I gives a thorough and systematic review of the fundamental processes with whole numbers, common fractions, and decimals. It unifies the teaching of the processes. The fundamental principles underlying the processes are discussed, and abundant drill exercises are furnished through which the student may become efficient in arithmetical computation.

Part II discusses the applications of arithmetic to business, under four heads, viz.: Accounts; Buying and Selling; Borrowing, Loaning, and Investing Money; and Cancelling Indebtedness. This part is sufficiently complete for the needs of any kind of school giving a course in commercial arithmetic. It sets forth the details of present commercial practices in such a way that the boy who studies it should not have to unlearn his arithmetic when he takes up commercial work.

Part III discusses certain advanced processes, including approximations and abridged processes, powers and roots, logarithms, principles in mensuration and proportion, with their applications to practical problems of an industrial character and to problems in the sciences. With the drill in the processes here given, the student is prepared to make any numerical computation that he will have occasion to make in any of the trades or in a scientific pursuit.

Part IV consists of a supplementary list of vocational problems. These are problems in agriculture, the building trades, shop work, etc. While these problems are not classified, those of

the various types are arranged consecutively, so that it will be easy for teachers to select those problems best suited for each particular class of students.

While we cannot mention all who have so kindly contributed to the preparation of this book, we feel that public recognition is due to W. B. Russell, Asst. Supt. Apprentices, New York Central Lines, and to Arthur D. Dean, Supervisor of Evening Schools, Massachusetts and Rhode Island Y. M. C. A., for valuable assistance rendered in gathering real industrial problems; to Supt. E. C. Warriner, Saginaw, Mich., for the use of material gathered during his recent visit to the vocational schools of Germany; and to H. N. Chute, co-author of the Carhart and Chute Physics, for reading the proof and offering many helpful suggestions.

July 15, 1908.

J. C. S.

J. F. M.

SUGGESTIONS TO TEACHERS

1. Purpose of arithmetic. The purpose of an advanced course in arithmetic is to fix in the mind of the student the fundamental principles upon which our methods of computation depend, and to show the applications of the subject in the various vocations. The problems should be *real*, i.e., such as are actually met in the different vocations in life. While a teacher should seek problems from different sources that fit the needs of his class, he should not waste the time of the student with the solutions of puzzles.

2. Degree of accuracy. Carrying the computation of a quantity further than the actual demands of the industrial or commercial world require should be discouraged. Thus, the cost of goods to \$.001, the volume of a cellar to .1 cu. ft., the lumber required for a house to .1 ft., etc., should not be allowed. Likewise, carrying out a result further than the accuracy of the data warrant is wrong. (See § 114.)

3. Estimating results. In all solutions of problems, either an estimate of the result should be made before the actual computation is begun, or the question "Does the result seem reasonable?" should be asked when the computation has been finished. This will usually check all such errors as the wrong position of the decimal point. (See § 48.)

4. Checks. In actual life one must know that results are *right*, or his arithmetic is of no value to him. *Accuracy in*

computation cannot be overemphasized. While the *method* of solving a problem cannot be checked, but depends upon the judgment of the student, all *computation* can and should be checked. *Students should form the habit of checking their work at every step.*

5. Methods of checking. *Addition* can be checked by performing the work again in a different order, as adding up, then down; *subtraction*, by adding the difference to the subtrahend; *multiplication*, by interchanging multiplier and multiplicand, or by multiplying in turn by the factors of the multiplier; *division*, by taking the product of the divisor and quotient, and adding the remainder; *square root*, by squaring the result.

In all abridged processes the work is best checked by carefully performing it a second time.

Insist upon all operations of all the problems of the course being performed twice.

SECONDARY ARITHMETIC

PART I

THE PROCESSES OF ARITHMETIC

CHAPTER I.

NOTATIONS.

1. The origin of number. The idea of number must have existed long before a symbolism by which to express it. In fact, the number idea must have been coeval with the human race. With the ability to distinguish differentness, that *this* is not *that*, must have originated the concept of number. Number answers the questions *how many*, found by counting distinct objects, or *how much*, found by measuring one quantity by another taken as a unit.

2. Symbols for expressing number. As the human race developed and greater use was made of number, a desire naturally arose for a means of expressing it. The characters used must have been as varied as the tribes or nations themselves. The first methods of any nation, however, must have been some *one-to-one correspondence*. That is, a sign of some sort, as a stroke in the sand, a pebble, a finger, or some other sign, must have been used to correspond with each individual thing of the group to be represented.

The student is familiar with the Roman and the Hindu

symbols. The Babylonians used a single wedge-shaped symbol. By giving it different positions it stood for 1, 10, and 100, respectively. The Egyptians used hieroglyphics and hieratics to represent 1, 10, and various powers of 10; the Greeks used the letters of their alphabet.

3. The principles by which other numbers were formed. The Romans used the *additive*, *subtractive*, and *multiplicative* principles. Thus, $II=I+I=2$; $VI=V+I=6$; $IV=V-I=4$; $IX=X-I=9$; $\overline{X}=1000 \times X=10,000$; $\overline{C}=1000 \times C=100,000$.

The Babylonians used the *additive* and *multiplicative* principles, while the Egyptians used but the *additive*.

In general, it may be said that the general plan of all the older systems of notation was much like that of the Roman system.

4. The defects of these systems. It is easily seen that these systems of notation served merely the first need of a notation, namely, a means of expressing a number idea. For purposes of computation they were clearly defective. This is seen by trying to find the product of any two numbers expressed by the Roman system. Nations using these systems performed all computations on some kind of *abacus*, of which the *swanpan* used by the Chinese today is an example.

5. The Hindu system of notation. We use the Hindu system of notation. We do not know when its development began nor how long it was in growing to its present perfection. Perhaps the nine numerals without the zero, not exactly in their present form, were used as early as the second century, A. D. The introduction of zero, without which the *place value feature* would have been impossible, likely came much later, perhaps during the fifth, sixth, seventh, or even eighth century A. D. This notation was known in Europe as early as the eleventh or twelfth century, but little use was made of it before the fifteenth century.

The *place-value* feature makes this system unique. By this system, each character may represent two values, an *intrinsic value* depending upon its shape, and a *place value* depending upon its position. *It is a decimal, place-value system of notation.*

Thus, the 4's of \$444 represent different values, each one just 10 times the one to the right of it.

6. The advantages of the Hindu notation. The place value feature allows a number to be broken into parts and dealt with, a part at a time. However large the numbers with which we are dealing, the real work in addition is the repetition of adding a number less than 10 to one less than 100. In multiplication, besides uniting the partial products, the real work is a repetition of finding products less than 10×10 . Thus, to find 389×496 we never need to consider products larger than 9×9 . Herein lies the great advantage of the Hindu system.

7. The notation of a common fraction. The primary *integral* unit is 1. By the place-value feature of our notation for integers, the relation of any number to 1 is shown by the position of the figures that denote the number. This is not so in common fractions. In the fraction $\frac{3}{4}$, 3 is the number of which we are thinking, but 4 shows the relation of this number to 1. The 4, then, may be considered the *scale* of the fraction, for it shows by what the number must be multiplied to give three 1's.

Then the number above the line (the numerator) denotes the number of things (units), and the number below the line (the denominator) denotes the size of these things by showing their relation to the primary integral unit. That is, the denominator shows the number of parts into which the primary integral unit has been divided.

8. A fractional unit. A fraction that denotes a single thing,

or one of the single parts into which the primary integral unit has been divided, is a *fractional unit*.

Thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, etc., are *fractional units*. Tell the scale of each, that is, the number of each that it will take to make 1.

9. The notations of fractions and integers compared. In integers, the value of a number denoted by a given figure *depends upon the place it occupies*. In fractions, the value *depends upon the denominator*.

Thus, the 5 of 50, 500, 5000, $\frac{5}{10}$, and $\frac{5}{100}$, represents different values in each.

10. Notation of a decimal fraction. When the denominator of a fraction is 10, or some power of 10, the fraction may be written in another form, the unit being expressed by the *position* of the numerator.

Thus, in 8.5 we know that the unit of which 5 is the number is just $\frac{1}{10}$ of the unit of which 8 is the number. Then, since 8 denotes 1's, 5 must denote 10ths. In 4.65, since 4 denotes 1's, 6 denotes 10ths and 5 denotes 100ths, or 65 denotes 100ths.

The **decimal point** is used to locate ones' place. That is, the figure to the left of the decimal point denotes the number of 1's.

NOTE. It is easily seen that with any scale those fractions whose denominators are powers of the scale may be expressed in the form of our decimal fractions. Thus, in scale 5, $4 + \frac{3}{5} + \frac{2}{25} = 4.32$.

11. Origin of a decimal base. It may have been noticed that in the Babylonian, Egyptian, and other systems of notation discussed, each system seems to have been based upon 10. That is, the characters suggest that the counting was done by grouping 10's or powers of 10. The Roman system may more truly be said to be based upon 5 and 10. This grouping of 10's or powers of 10 evidently did not depend upon any essential principle of the notations themselves but doubtless upon the habits of man in counting the fingers and representing

numbers by the fingers. Had the number of fingers been different, the base or scale of the number systems of the world would most surely have been different.

12. The requirements of a convenient base. A base of any convenient size may be used in a system making use of place value. The number of characters required is one less than the base, together with a character to denote the absence of value.

Thus, with a base of 5, eight is expressed by 13, *i. e.*, $1 \times 5 + 3$; ten by 20, *i. e.*, $2 \times 5 + 0$; sixteen by 31; thirty-six by 121; etc.; and to express any number with the base 5, only the characters 1, 2, 3, 4, and 0 are needed.

With a base of 10 all operations can be performed upon numbers of any size by knowing the 36 combinations in multiplication from 2×2 to 9×9 , and the 45 combinations in addition from $1 + 1$ to $9 + 9$. (§6.)

With a base of 5, there are but 6 combinations needed in multiplication, and 10 in addition. With 12 as a base, 55 combinations are needed in multiplication and 66 in addition. Thus it is seen that the larger the base the more facts there are to remember, while the smaller the base the more figures there are needed to express a number, especially a large number.

13. The bases 10 and 12 compared. Either 10 or 12 is a base of convenient size. A larger base would require too much memory work. A very much smaller base would require too many figures to express a large number.

Thus, with a scale of 15, there are 120 combinations in addition and 105 in multiplication. With a scale of 5, 9684 is expressed by 302214.

As to the bases 12 and 10, 12 has factors 2, 3, 4, and 6; 10 has but 2 and 5. Hence, many of the common fractions which occur most frequently in business may be expressed more simply in terms of 12ths, 144ths, etc., than in terms of 10ths, 100ths, etc.

or one of the single part unit has been divided, is

Thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc., and that is, the number of each

9. The notations of integers, the value of a number upon the place it occupies the denominator.

Thus, the 5 of 50, 500, 5000 each.

10. Notation of a decimal of a fraction is 10, or written in another form position of the number

Thus, in 8.5 we know $\frac{5}{10}$ of the unit of which 8 denote 10ths. In 4.65, 100ths, or 65 denotes 100

The decimal point figure to the left of of 1's.

NOTE. It is easily seen that inators are powers of decimal fractions. The

11. Origin of a decimal that in the Babylonian notation discussed, each That is, the character by grouping 10's and more truly be said of 10's or powers essential principle upon the habits of

AMPLE 2. Change 4265_8 to base 10.

It equals $4 \times (8 \times 8 \times 8) + 2 \times (8 \times 8) + 6 \times 8 + 5$ or 2229_{10} . It may be quickly as follows :

$$\begin{array}{r} 4265_8 \\ \underline{8} \\ 34 \\ \underline{8} \\ 278 \\ \underline{8} \\ 2229 \end{array}$$

EXPLANATION. Beginning with the highest order and multiplying by 8, (expressing the result in base 10) and adding 2, we have 34. Multiplying 34 by 8 and adding 6, we have 278, etc.

AMPLE 3. Change 4324_5 to scale 8.

EXPLANATION. This is the method of Example 1. We divide by 8, which in the scale of 5 is 13, keeping the remainders. $43_5 + 13_5 = 2$ and 12_5 remaining ; $122_5 + 13_5 = 4$ (for it is $37 + 8$) and 10_5 remaining ; $104_5 + 13_5 = 3$ and 10_5 , or 5, remaining. Show how the remaining divisions are formed.

Since $4324_5 = 1115_8$.

NOTE. If this seems too difficult, it may be omitted without interfering with subsequent work. The "second method" is more simple.

Second Method.

$$\begin{array}{r} 4324_5 \\ \underline{5} \\ 27 \\ \underline{5} \\ 165 \\ \underline{5} \\ 1115 \end{array}$$

EXPLANATION. Here we use the method of Example 2, but express the products in 8's instead of in 10's. Thus, $5 \times 4 + 3 = 23$, but this is $2 \times 8 + 7$ or 27. Again $5 \times 7 + 2 = 37$, but this is $4 \times 8 + 5$ or 45. $5 \times 2 + 4$ (of the 45) = 14 or $1 \times 8 + 6$, that is 16. Show the rest of the work.

EXERCISES.

1. What does 3425_8 , scale 8, mean? What would it mean in scale 12?
2. What two characters are needed in a scale of 2? Express 45 in a scale of 2.
3. What characters are needed in a scale of 5? In a scale

of 8? In a scale of 12? Express 3426 in a scale of 5. In a scale of 8. In a scale of 12.

NOTE. In a scale of 12, let t stand for ten, and e eleven.

4. Change 3224₆ to a scale of 8 in two ways.
5. Change 3122₆ to a scale of 12 in two ways
6. Change 1101110₂ to a scale of 5 in two ways.
7. Change 93468₁₂ to a scale of 5 in two ways.
8. Use each method to change 32432₆ to our scale.
9. Use each method to change 34625₆ to our scale.
10. Use each method to change 594 t 2₁₂ to our scale.

CHAPTER II.

ADDITION.

15. Fundamental principle. The fundamental principle of addition is that *only like things, or things considered like, may be added.*

16. The use made of our notation. By the *place-value* feature of our notation, we know which figures of several numbers represent like things, as ones, tens, etc. It is more convenient in addition if figures representing like numbers are written in vertical columns.

17. Accuracy and rapidity. Since accuracy and rapidity depend upon the instant and accurate recognition of the 45 primary number facts in addition, the following tables should be used for frequent drill.

DRILL TABLES.

	1	2	3	4	5
A	2 5 8	6 2 7	9 5 1	2 9 5	6 1 2
B	3 6 3	9 1 7	4 5 3	3 5 4	3 1 8
C	7 1 6	3 6 1	2 8 7	7 1 8	4 5 3
D	4 9 7	3 6 7	6 4 3	8 4 8	2 1 5
E	9 6 4	8 7 2	9 5 6	7 8 3	4 7 2
F	2 8 6	1 5 2	3 8 1	2 9 4	4 9 5

NOTE. Adding the columns of each of the rectangles gives the 45 primary combinations. The rows of each rectangle will give drill work in adding three numbers, while drill in adding 4 or 6 numbers may be had from the columns of the whole diagram.

WRITTEN DRILLS.

Add downward and check by adding upward :

1	2	3	4	5	6	7
2056	8634	5914	2945	6114	9689	4268
8879	1988	6432	6847	7836	4592	3984
1483	2463	5696	9638	3971	8463	5963
2961	9574	8341	7436	8421	9857	8873
3484	8367	1963	8963	1635	7389	6984
9688	2931	4376	4786	1418	6473	5463
8746	4251	6184	3942	3796	1439	8777
8245	8688	1437	1873	8432	2486	6321
9683	9784	9846	8643	4868	3245	8479
7347	2693	8419	9475	9783	1731	9638
<u>5284</u>	<u>1785</u>	<u>1786</u>	<u>1291</u>	<u>4785</u>	<u>9338</u>	<u>7347</u>

18. Recording partial additions. Accountants, especially if they are liable to interruptions, usually record the sum of each column as it is added. In this way an error is more easily found. The plan is as follows :

3463	
2974	
9362	51
4736	53
9265	42
7989	49
6287	<u>53781</u>
5239	
4461	
<u>53781</u>	

NOTE. If each column has been added downward, it is checked by adding upward.

When using this method, we may begin adding with any column.

A second method. A little time may be saved by adding the number to be carried before recording the sum of each column.

3462	26	sum of the second column with 2 (the tens from 26)
4578	27	added ; 32 is the sum of the third with 2 (from 27)
9634	32	added, etc. The sum is 27276.

2759	27	This method saves adding the several sums, but
<u>6843</u>		the columns must be added in order from right to
<u>27276</u>		left.

DRILL EXERCISES.

It is frequently desirable to add numbers written in horizontal lines as well as numbers written in columns.

1-7. Without rewriting, find the sum of each column below.

8-33. Without rewriting, find the sum of each row.

	1.	2.	3.	4.	5.	6.	7.
<i>a</i>	2	13	63	197	1345	46,768	104,876 <i>a</i>
<i>b</i>	9	28	80	928	2769	14,693	340,276 <i>b</i>
<i>c</i>	7	49	91	873	3876	24,768	950,073 <i>c</i>
<i>d</i>	5	15	64	417	4245	71,293	268,479 <i>d</i>
<i>e</i>	10	30	81	652	8947	60,037	728,735 <i>e</i>
<i>f</i>	3	54	93	765	9038	73,407	629,876 <i>f</i>
<i>g</i>	8	20	68	739	6705	19,386	724,894 <i>g</i>
<i>h</i>	11	32	84	984	9477	47,682	548,075 <i>h</i>
<i>i</i>	4	56	95	859	6841	86,573	829,386 <i>i</i>
<i>j</i>	12	21	70	487	7238	94,775	445,876 <i>j</i>
<i>k</i>	6	36	87	529	5467	89,238	377,872 <i>k</i>
<i>l</i>	15	57	96	843	3094	64,839	763,804 <i>l</i>
<i>m</i>	5	24	72	468	6293	29,466	689,903 <i>m</i>
<i>n</i>	9	42	89	757	4787	48,748	670,498 <i>n</i>
<i>o</i>	8	59	69	836	6432	98,396	988,875 <i>o</i>
<i>p</i>	4	25	75	793	8978	47,874	687,568 <i>p</i>
<i>q</i>	7	45	90	581	6479	56,832	994,693 <i>q</i>
<i>r</i>	11	48	98	629	9463	91,478	849,376 <i>r</i>
<i>s</i>	6	27	76	924	8779	16,923	649,473 <i>s</i>
<i>t</i>	13	60	99	786	3678	94,873	384,923 <i>t</i>
<i>u</i>	16	75	77	937	6482	73,689	569,247 <i>u</i>
<i>v</i>	19	46	93	724	5962	69,798	347,964 <i>v</i>
<i>w</i>	17	74	87	692	7389	54,991	976,394 <i>w</i>
<i>x</i>	7	69	79	496	8917	98,346	298,973 <i>x</i>
<i>y</i>	9	57	96	739	8998	79,819	917,966 <i>y</i>
<i>z</i>	14	93	67	496	9843	54,433	723,468 <i>z</i>

19. Adding two columns at a time. If one has a great deal of adding to do, time may be saved by becoming able to add two columns at a time. Much practice in written work of this

kind may not be worth while, but all students will find a great advantage in being able to add quickly any two numbers less than 100. The method of gaining this ability will be shown in the next section.

20. Oral drills. By using a permanent chart similar to the following, or the table itself for a few minutes each day, great proficiency may soon be gained.

First add tens, *i. e.*, 10, 20, 30, etc., up to 90, to each number in the table; then add ones, *i. e.*, 1, 2, 3, etc., up to 9, to the same numbers. When this can be done rapidly any two adjacent numbers, or any number and the one below it in the table, should be taken. To add 24 and 58, say 78, 82, adding first the *tens* then the *ones*.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	18	64	60	90	85	48	89	54	25	98	86	47	53
2	74	91	88	20	63	16	72	78	10	84	28	96	34
3	38	19	79	39	34	67	40	24	53	72	86	75	98
4	56	32	92	14	57	26	35	88	43	81	96	84	72
5	37	81	49	93	36	52	17	23	72	85	32	47	89
6	51	15	96	59	27	83	65	99	68	72	65	79	86
7	81	61	69	29	95	21	94	42	84	49	61	84	29
8	70	87	28	98	18	41	55	80	75	63	89	72	96
9	58	60	97	46	82	62	22	12	66	98	42	69	87
10	47	11	71	80	86	73	45	76	44	75	96	27	81

21. The general process of addition. The process of addition, as used in the preceding pages, is universal in its application, whatever the base of the system. That is, whatever be the relation of one unit to the next higher, the process is the same. The principle stated in § 15 is used in all addition.

EXAMPLE 1. Add 3.46, 4.986, 6.8, 5.26.

3.46	ones, (§ 16) by placing the points under each other
4.986	like units are placed in columns throughout. Since
6.8	the fractional parts at the right of the points have the same
5.26	decimal scale of relations, the number of tens in the sum
<u>20.456</u>	of each column is added to the next column, as in

integers.

EXAMPLE 2. Add 8 yd., 2 ft., 8 in.; 2 yd., 1 ft., 5 in.; 6 yd., 2 ft., 7 in.

(8)	(12)	EXPLANATION. Here the sum is 20 in the first
yd.	ft.	in.
8	2	8
2	1	5
6	2	7
<u>18</u>	<u>0</u>	<u>8</u>

column, but the scale is 12; that is, it takes 12 of these units (inches) to make 1 of the next higher (feet). Hence we divide 20 by 12, and have 1 ft. to add to the second column and 8 in. to write down. In the next column the sum is 6, but the scale is 3. So we have 2 to add to the third column and none to write down.

EXAMPLE 3. Add $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{5}$.

$\frac{1}{2}$	EXPLANATION. Here we are to add 3 things, 2 things,
$\frac{2}{3}$	and 4 things, and since they are all alike we have 9 things,
$\frac{4}{5}$	which are, in this case, 5ths. Hence the sum is $\frac{17}{15}$.
$\frac{1}{2}$ or $1\frac{1}{2}$	Now $\frac{1}{2}$ make an integral unit or 1. Hence $\frac{17}{15} = 1\frac{2}{15}$.

EXAMPLE 4. Let 3248, 4132, 3412, and 1844 represent numbers in the scale of 5. Add them.

3248	EXPLANATION. Here we have eleven in the first column,
4132	or two 5's and 1. Writing the 1, we add 2 to the next
3412	column, which gives fourteen or two 5's and 4. Writing
1844	4, we add 2 to the next column, which gives 12 or two 5's
<u>23241</u>	and 2, etc.

DRILL EXERCISES.

Add the following:

1. 3 ft., 6 in.; 8 ft., 9 in.; 10 ft., 7 in.; 8 ft., 4 in.; 14 ft., 10 in.; 18 ft., 11 in.

2. $6^{\circ} 30' 45''$; $12^{\circ} 29' 46''$; $18^{\circ} 45' 23''$; $46^{\circ} 28' 36''$; $18^{\circ} 46''$; $17^{\circ} 34'$.

3. 84.6; 28.49; 6.845; 17.3; 96.81; 109.385; 97.68; 9.763.

$$4. \frac{8}{8}; \frac{9}{8}; \frac{6}{8}; \frac{7}{8}; \frac{5}{8}; \frac{11}{8}; \frac{1}{8}.$$

$$5. \frac{3}{7}; \frac{5}{7}; \frac{9}{7}; 1\frac{6}{7}; \frac{34}{7}; \frac{2}{7}.$$

$$6. 3443; 2343; 2434; 4231; 2423; 1441; 2434; \text{base } 5.$$

$$7. 3542; 5637; 3437; 6735; 7326; 3437; 5364; 2634; \text{base } 8$$

$$8. 932t; tee9; 1ete; 8t54; 7e45; 6t3e; 1e57; eett; \text{base } 12.$$

NOTE. t = ten; e = eleven.

$$9. 46.23; 54.37; 63.42; 76.265; 34.361; 36.26; \text{base } 8.$$

$$10. 101.101; 11.011; 111.11; 10.111; 100.11; 10.001; \text{base } 2.$$

$$11. e.t39; t0.e13; 9t.ee; 7e.e5; 6t.te; 54.tee; \text{base } 12.$$

$$12. 34.2135; 22.315; 30.2135; 14.045; 11.445; \text{base } 10.$$

$$13. 175.et; ee.t5; t51.t2; 69.9e1; 73.9t; \text{base } 12.$$

22. Changing the units of a fraction. The student already understands the notation of a fraction (§ 7). That is, the number written above the line (the *numerator*) denotes *the number of things*; and the number below the line (the *denominator*) shows what these things are by showing into *how many parts* the integral unit has been divided.

Multiplying both terms (numerator and denominator) of a fraction by the same number does not change the value.

For multiplying the denominator by any number, say 5, gives a unit only one-fifth as large, while multiplying the numerator by the same thing gives 5 times as many of these units, which evidently gives a new fraction equivalent to the old one.

Since a fraction may be changed to an equivalent one with larger terms by introducing the same factor in each term, it naturally follows that:

A common factor may be removed from both terms of a fraction without changing the value of the fraction.

$$\text{Thus, } \frac{2}{3} = \frac{5 \times 2}{5 \times 3}, \text{ or } \frac{10}{15}; \frac{12}{20} = \frac{4 \times 3}{4 \times 5} = \frac{3}{5}.$$

23. Changing fractions to like units. Since a fraction may

be changed to an equivalent one whose denominator is any given *multiple* of the given denominator by the method of §22, it follows that to change two or more fractions to a common unit, we must first find a *common multiple* of the given denominators, that is, *a number that is divisible by each of them*. Usually this can be found by inspection in the case of all fractions that occur in practical work.

24. To find the least common multiple. While one rarely ever needs a multiple that cannot be seen by inspection, the following examples illustrate a method that may be used where the multiple cannot readily be seen.

EXAMPLE 1. Find the *least common multiple* (*L. C. M.*) of 30 and 42, *i. e.*, the least number that is divisible by each of them.

$$30 = 2 \times 3 \times 5.$$

$$42 = 2 \times 3 \times 7.$$

EXPLANATION. A multiple of 30 must contain all the prime factors of 30, and a multiple of 42 all the prime factors of 42. Now, a number whose factors are 2, 3, 5, and 7, will contain $2 \times 3 \times 5$, or 30, and also $2 \times 3 \times 7$, or 42. In practical work we say 30×7 , or 42×5 , since 7 is not found in 30, or 5 in 42.

EXAMPLE 2. Find the *L. C. M.* of 60, 72, and 108.

$$60 = 2 \times 2 \times 3 \times 5.$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3.$$

$$108 = 2 \times 2 \times 3 \times 3 \times 3.$$

$$108 \times 2 \times 5 = 1080, \text{ the L. C. M. in the L. C. M., is not found in } 108?$$

EXPLANATION. We ask ourselves the following questions :

(a) What prime factor of 60, needed

in the L. C. M., is not found in 108?

(b) What prime factor of 72 is not found in 108, *i. e.*, found more times in 72 than in 108?

(c) What is meant by the least common multiple of several numbers?

DRILL EXERCISES.

Add the following :

1. $\frac{7}{18}, \frac{4}{21}.$

2. $\frac{7}{18}, \frac{11}{24}.$

3. $\frac{3}{8}, \frac{17}{18}.$

4. $\frac{7}{24}, \frac{11}{21}.$

5. $\frac{29}{48}, \frac{29}{32}.$

6. $\frac{3}{7}, \frac{5}{12}, \frac{23}{42}, \frac{16}{21}.$

7. $\frac{7}{40}, \frac{17}{30}, \frac{61}{80}, \frac{41}{80}.$

8. $\frac{5}{36}, \frac{19}{32}, \frac{17}{16}, \frac{19}{48}.$

9. $13\frac{1}{8}, 24\frac{1}{4}, 37\frac{1}{12}, 41\frac{2}{3}.$

Add :

10.	11.	12.	13.	14.	15.	16.
$12\frac{1}{2}$	$11\frac{1}{5}$	$16\frac{1}{4}$	$32\frac{5}{8}$	$82\frac{1}{2}$	$15\frac{1}{2}$	$22\frac{1}{2}$
$6\frac{1}{10}$	$9\frac{1}{2}$	$24\frac{1}{4}$	$84\frac{1}{2}$	$24\frac{1}{2}$	$23\frac{1}{2}$	$86\frac{1}{2}$
$8\frac{1}{2}$	$7\frac{1}{2}$	$42\frac{1}{2}$	$48\frac{1}{2}$	$86\frac{1}{2}$	$18\frac{1}{2}$	$86\frac{1}{2}$
$6\frac{7}{8}$	$5\frac{1}{2}$	$88\frac{1}{2}$	$34\frac{1}{2}$	$32\frac{1}{2}$	$32\frac{7}{8}$	$42\frac{1}{2}$
$1\frac{1}{2}$	$9\frac{1}{4}$	$92\frac{1}{4}$	$52\frac{1}{2}$	$84\frac{1}{2}$	$84\frac{1}{2}$	$43\frac{5}{8}$
$9\frac{1}{2}$	$8\frac{1}{2}$	$26\frac{1}{2}$	$69\frac{1}{2}$	$32\frac{1}{2}$	$46\frac{1}{2}$	$23\frac{1}{2}$
$8\frac{1}{2}$	$7\frac{1}{2}$	$84\frac{1}{2}$	$82\frac{1}{2}$	$18\frac{1}{2}$	$53\frac{1}{2}$	$48\frac{1}{2}$

DRILL TABLE FOR ORAL WORK.

NOTE. $a+b$ indicates that the adjacent fractions in columns a and b are to be added.

	a	b	c	d	e	f	g	h
1.	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{10}$	$4\frac{1}{2}$	$12\frac{1}{2}$	$19\frac{1}{2}$	$341\frac{1}{2}$
2.	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{10}$	$6\frac{1}{2}$	$16\frac{1}{2}$	$19\frac{1}{5}$	$482\frac{1}{10}$
3.	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{2}$	$8\frac{1}{2}$	$18\frac{1}{2}$	$16\frac{1}{2}$	$386\frac{1}{2}$
4.	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{2}$	$\frac{1}{5}$	$7\frac{1}{5}$	$31\frac{1}{2}$	$24\frac{1}{2}$	$521\frac{1}{2}$
5.	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{5}$	$8\frac{1}{2}$	$33\frac{1}{2}$	$24\frac{1}{2}$	$287\frac{1}{2}$
6.	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{10}$	$9\frac{1}{2}$	$37\frac{1}{2}$	$41\frac{1}{2}$	$272\frac{1}{2}$
7.	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{2}$	$1\frac{1}{2}$	$6\frac{1}{2}$	$43\frac{1}{2}$	$86\frac{1}{2}$	$316\frac{7}{10}$
8.	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$7\frac{7}{10}$	$56\frac{1}{2}$	$45\frac{1}{2}$	$332\frac{1}{2}$
9.	$\frac{1}{10}$	$1\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{2}$	$10\frac{1}{2}$	$62\frac{1}{2}$	$48\frac{1}{10}$	$681\frac{1}{2}$
10.	$\frac{1}{10}$	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{5}$	$11\frac{1}{2}$	$66\frac{1}{2}$	$52\frac{1}{2}$	$218\frac{1}{2}$

Suggestive exercises :

1-10.	$a+b$.	41-50.	$f+g$.	81-90.	$g+h$.
11-20.	$a+c$.	51-60.	$e+h$.	91-100.	$c+g$.
21-30.	$a+d$.	61-70.	$e+f$.	101-110.	$a+b+c$.
31-40.	$b+c$.	71-80.	$d+f$.	111-120.	$b+c+d$.

CHAPTER III.

SUBTRACTION.

25. Methods of subtraction. Since subtraction is an inverse process, it naturally depends upon the direct process, *i. e.*, *addition*.

There are two general methods of subtraction now in use. They are both shown below.

$$\begin{array}{r} 932637 \\ 156392 \\ \hline 776245 \end{array}$$
First method. In this we think, 2 from 7, 5; 9 from 13 (1 of the 6 making 10 of the order of the 3), 4; 3 from 5 (1 of the 6 has been used), 2; 6 from 12, 6; 5 from 12, 7; 1 from 8, 7.

Second method. In this method, called the *Austrian method*, we think, 2 and 5 are 7; 9 and 4 are 13; 1 (from the 10 of 13) and 3 and 2 are 6; 6 and 6 are 12; 1 and 5 and 7 are 13; 1 and 1 and 7 are 9; writing, as we thus add, the number indicated in the full-faced type.

DRILL EXERCISES.

Subtract:

1. $\begin{array}{r} 340,639 \\ 184,793 \\ \hline \end{array}$	3. $\begin{array}{r} 503,164 \\ 198,756 \\ \hline \end{array}$	5. $\begin{array}{r} 86,735,421 \\ 28,306,843 \\ \hline \end{array}$	7. $\begin{array}{r} 90,807.00 \\ 18,584.68 \\ \hline \end{array}$
2. $\begin{array}{r} 716,884 \\ 197,869 \\ \hline \end{array}$	4. $\begin{array}{r} 84,693,705 \\ 19,906,987 \\ \hline \end{array}$	6. $\begin{array}{r} 24,180,250 \\ 18,796,784 \\ \hline \end{array}$	8. $\begin{array}{r} 32,090.100 \\ 1,846.965 \\ \hline \end{array}$
9. $\begin{array}{r} 346,893,463 \\ 193,906,847 \\ \hline \end{array}$	10. $\begin{array}{r} 807,632,947 \\ 198,469,758 \\ \hline \end{array}$	11. $\begin{array}{r} 100,968,482 \\ 97,896,745 \\ \hline \end{array}$	

26. Subtracting by adding arithmetical complements. 8—3 is the same as 8+10—3—10, for 10 is both added and sub-

tracted in the former to get the latter. This may be written $8+(10-3)-10$, or $8+7-10$. Or in general, $a-b=a+10-b-10=a+(10-b)-10$.

Hence, we may subtract a number by adding the *complement*, or the difference between the number and 10, if we drop 10 from the sum.

EXAMPLE 1. $8-3+4-6=8+7+4+4-20=3$.

This method is most useful in balancing numbers written in lines where some are to be subtracted.

EXAMPLE 2.

BALANCE.	CHECKS PAID.	DEPOSITS.	BALANCE.
396.40	283.20	342.96	456.16
265.73	192.84	162.87	235.26

EXPLANATION. Here we have in the first line 6; then 9, 17 (adding 8, the complement of 2), 21. Carrying 1 (1 of the 2 tens was dropped), 3, 10, 16; 4, 6, 15; 3, 11, 14; writing those in full-faced type.

In the second we have 7, 13, 16; 3, 5, 12; 2, 10, 15; 6, 7, 13; 1, 10, 12.

DRILL EXERCISES.

Find the balance in each line:

	BALANCE.	CHECKS PAID.	DEPOSITS.	BALANCE.
1.	\$346.81	\$297.68	\$143.82	
2.	576.43	438.63	346.25	
3.	964.27	724.31	287.38	
4.	348.93	196.38	395.50	
5.	964.22	896.34	483.96	
6.	314.96	125.38	465.80	
7.	792.04	96.42	345.21	
8.	342.84	85.63	236.95	
9.	938.14	125.46	95.60	

	BALANCE.	CHECKS PAID.	DEPOSITS.	BALANCE.
10.	\$890.42	\$735.19	\$483.42	
11.	765.48	98.18	385.62	
12.	342.96	97.42	196.84	
13.	792.05	346.81	560.80	
14.	309.17	85.80	190.80	
15.	806.24	190.60	265.85	
16.	973.61	385.42	463.73	
17.	843.42	97.86	196.85	
18.	763.96	119.43	234.25	
19.	834.42	380.40	198.17	
20.	<u>765.25</u>	<u>195.98</u>	<u>380.42</u>	<u>.....</u>

21. Find the sum of the balances in the first column, the sum of the twenty checks, the sum of the deposits, and the sum of the required column of balances. The sum of the first balance and the deposits, less the sum of the checks paid, should equal the sum of the required balances. This will be a *check* of your work.

27. The general process of subtraction. The methods shown in § 25 may be used with any scale of relation.

EXAMPLE 1. From 9 yd., 2 ft., 3 in., take 2 yd., 2 ft., 8 in.

	(8)	(12)	EXPLANATION.
yd.	ft.	in.	Since 8 cannot be taken from 3, add one of the 2 feet to the 3 inches, making 15 inches. 8 in. from 15 in. leaves 7 in. 2 from 1 cannot be taken. Hence, one of the 9 yd. is added to the 1 ft., making 4 ft. Then 2 ft. from 4 ft. leaves 2 ft., etc.
9	2	3	
2	2	8	
6	2	7	

EXAMPLE 2. From 3465, take 4234.

8)4234,	3465,	be subtracted until both are changed to the same scale. Instead of changing 4234, to scale 8, 3465, might have been changed to scale 5, or both might have been changed to scale 10, or to any other scale.
8)241—1	1071,	
8)13—7	2374,	
1—0		

EXAMPLE 3. From $48\frac{1}{2}$ take $24\frac{3}{4}$.

$48\frac{1}{2}$ **EXPLANATION.** Changing $\frac{1}{2}$ and $\frac{3}{4}$ to like units, we have $\frac{2}{4}$
 $24\frac{3}{4}$ and $\frac{3}{4}$. Taking $\frac{3}{4}$ from $1\frac{1}{2}$ (the 1 taken from the 8), we have
 $28\frac{2}{4}$ $\frac{2}{4}$; 4 from 7, 3; 2 from 4, 2.

DRILL EXERCISES.

Subtract:

$$\begin{array}{r} 1. \quad 321468234291 \\ \underline{186983492618} \end{array}$$

$$\begin{array}{r} 3. \quad 54001324689842 \\ \underline{19683249869825} \end{array}$$

$$\begin{array}{r} 2. \quad 50032406983 \\ \underline{18342983246} \end{array}$$

$$\begin{array}{r} 4. \quad 81262419630824 \\ \underline{19638142986814} \end{array}$$

$$\begin{array}{r} 5. \quad 215\frac{1}{2} \\ \underline{98\frac{3}{8}} \end{array}$$

$$\begin{array}{r} 6. \quad 342\frac{1}{4} \\ \underline{196\frac{7}{8}} \end{array}$$

$$\begin{array}{r} 7. \quad 297\frac{2}{3} \\ \underline{125\frac{1}{2}} \end{array}$$

$$\begin{array}{r} 8. \quad 368\frac{1}{8} \\ \underline{124\frac{5}{12}} \end{array}$$

$$\begin{array}{r} 9. \quad 347\frac{1}{4} \\ \underline{189\frac{3}{8}} \end{array}$$

$$\begin{array}{r} 10. \quad 321\frac{1}{4} \\ \underline{96\frac{3}{4}} \end{array}$$

$$\begin{array}{r} 11. \quad 342\frac{1}{8} \\ \underline{135\frac{5}{12}} \end{array}$$

$$\begin{array}{r} 12. \quad 968\frac{1}{4} \\ \underline{126\frac{1}{8}} \end{array}$$

$$\begin{array}{r} 13. \quad 26 \text{ lb. } 8 \text{ oz.} \\ \underline{12 \text{ " } 10 \text{ "}} \end{array}$$

$$\begin{array}{r} 14. \quad 34^{\circ} 12' 16'' \\ \underline{10^{\circ} 43' 38''} \end{array}$$

$$\begin{array}{r} 15. \quad 97 \text{ yr. } 6 \text{ mo. } 12 \text{ da.} \\ \underline{83 \text{ " } 9 \text{ " } 17 \text{ "}} \end{array}$$

Subtract at sight:

$$\begin{array}{r} 16. \quad 215\frac{5}{8} \\ \underline{39\frac{1}{2}} \end{array}$$

$$\begin{array}{r} 22. \quad 112\frac{1}{4} \\ \underline{46\frac{2}{3}} \end{array}$$

$$\begin{array}{r} 28. \quad 3059\frac{1}{4} \\ \underline{126\frac{5}{8}} \end{array}$$

$$\begin{array}{r} 34. \quad 408\frac{1}{12} \\ \underline{231\frac{1}{8}} \end{array}$$

$$\begin{array}{r} 17. \quad 160\frac{2}{3} \\ \underline{49\frac{11}{12}} \end{array}$$

$$\begin{array}{r} 23. \quad 109 \\ \underline{26\frac{1}{12}} \end{array}$$

$$\begin{array}{r} 29. \quad 2930\frac{1}{8} \\ \underline{427\frac{5}{6}} \end{array}$$

$$\begin{array}{r} 35. \quad 426\frac{1}{3} \\ \underline{108\frac{1}{8}} \end{array}$$

$$\begin{array}{r} 18. \quad 203\frac{5}{8} \\ \underline{98\frac{7}{8}} \end{array}$$

$$\begin{array}{r} 24. \quad 1096\frac{3}{8} \\ \underline{204\frac{3}{4}} \end{array}$$

$$\begin{array}{r} 30. \quad 734\frac{11}{16} \\ \underline{246\frac{7}{8}} \end{array}$$

$$\begin{array}{r} 36. \quad 117\frac{3}{8} \\ \underline{96\frac{7}{8}} \end{array}$$

$$\begin{array}{r} 19. \quad 106\frac{1}{8} \\ \underline{94\frac{11}{12}} \end{array}$$

$$\begin{array}{r} 25. \quad 4095\frac{2}{3} \\ \underline{738\frac{11}{12}} \end{array}$$

$$\begin{array}{r} 31. \quad 824\frac{1}{4} \\ \underline{163\frac{3}{5}} \end{array}$$

$$\begin{array}{r} 37. \quad 213\frac{5}{8} \\ \underline{108\frac{1}{4}} \end{array}$$

$$\begin{array}{r} 20. \quad 104\frac{1}{6} \\ \underline{26\frac{2}{3}} \end{array}$$

$$\begin{array}{r} 26. \quad 4152\frac{4}{5} \\ \underline{136\frac{2}{3}} \end{array}$$

$$\begin{array}{r} 32. \quad 610\frac{5}{9} \\ \underline{138\frac{11}{12}} \end{array}$$

$$\begin{array}{r} 38. \quad 261\frac{1}{3} \\ \underline{107\frac{5}{6}} \end{array}$$

$$\begin{array}{r} 21. \quad 1546\frac{2}{3} \\ \underline{119\frac{5}{8}} \end{array}$$

$$\begin{array}{r} 27. \quad 4012\frac{1}{6} \\ \underline{107\frac{7}{8}} \end{array}$$

$$\begin{array}{r} 33. \quad 341\frac{1}{11} \\ \underline{103\frac{1}{2}} \end{array}$$

$$\begin{array}{r} 39. \quad 230\frac{2}{3} \\ \underline{103\frac{5}{8}} \end{array}$$

SUBTRACTION

21

Subtract the following :

- | | | |
|---------------------------------|-----------------------------------|-------------------------|
| 40. $23413_5 - 14244_5$. | 46. $61302_8 - 35234_8$. | 52. $43.9 - 28.975$. |
| 41. $30411_6 - 14422_6$. | 47. $73421_{10} - 73421_8$. | 53. $96.87 - 57.909$. |
| 42. $63075_8 - 27367_8$. | 48. $34261_{12} - 34261_{10}$. | 54. $16.841 - 9.6342$. |
| 43. $31620_8 - 17654_8$. | 49. $ete001_{12} - 990t9t_{12}$. | 55. $.0346 - .013965$. |
| 44. $e30t2_{12} - 939t7_{12}$. | 50. $35.647 - 19.896$. | 56. $1.0368 - .95$. |
| 45. $43241_6 - 5321_6$. | 51. $3.401 - 1.9847$. | 57. $3.126 - .8436$. |

CHAPTER IV.

MULTIPLICATION.

28. Multiplying by an integer. It has been seen that an integral multiplier denotes how many times the multiplicand is to be taken as an addend.

Thus, $5 \times \$345$ means the same as $\$345 + \$345 + \$345 + \$345 + \$345$.

To multiply by a number of more than one figure the following principles are used :

Principle I. *To multiply a number by the sum of two or more numbers, we multiply the multiplicand by each of the numbers and add the several products.*

Thus, $25 \times 36 = (20 + 5) \times 36 = 20 \times 36 + 5 \times 36$.

Principle II. *One number may be multiplied by another by multiplying by each of its factors in succession.*

Thus, $6 \times 5 = 2 \times 3 \times 5$; $20 \times 14 = 2 \times 10 \times 14 = 2 \times 140$.

EXAMPLE 1. Multiply 86 by 27.

$$\begin{array}{r} 86 \\ 27 \\ \hline 602 = 7 \times 86 \\ 1720 = 20 \times 86 \text{ or } 2 \times 10 \times 86 \\ \hline 2322 = 27 \times 86 \end{array}$$

29. Special processes. By making use of the two principles of § 28, and by observing that the order of the addends does not affect the sum, work may sometimes be saved.

EXAMPLE 1. Multiply 273 by 856.

273	EXPLANATION. Observing that 56 is a mul-
<u>856</u>	multiple of 8, and that $56 \times 273 = 7 \times 8 \times 273$, we
218400	first find 800×273 . Then, 7×2184 will give
<u>15288</u>	56×273 . Adding, the total product is
233688	233,688.

EXAMPLE 2. Multiply 42683 by 72936.

42683	
<u>72936</u>	
33414700	$= 900 \times 42683$
1536588	$= 36 \times 42683$ How obtained ?
<u>3073176000</u>	$= 72000 \times 42683$ How obtained ? (Two ways.)
3113127288	

DRILL EXERCISES.

By the method of § 29, find the products:

- | | | |
|-----------------------|----------------------------|----------------------------|
| 1. 742×968 . | 6. 927×824 . | 11. 42847×37698 . |
| 2. 729×846 . | 7. 848×654 . | 12. 95463×72841 . |
| 3. 637×453 . | 8. 654×729 . | 13. 36972×84639 . |
| 4. 735×864 . | 9. 35742×87638 . | 14. 48966×79486 . |
| 5. 637×459 . | 10. 56872×97635 . | 15. 56728×39624 . |

30. The general process of multiplication. It would be inferred from the work in addition that as long as the meaning of multiplication does not change, the process must be the same, whatever scale is used.

EXAMPLE 1. Multiply 842 by 34, scale 5.

842	EXPLANATION. $4 \times 2 = 13$, that is $5 + 3$.
<u>34</u>	Write the 3 and carry the 1. $4 \times 4 + 1 = 32$,
3023	that is, $3 \times 5 + 2$. Write 2 and carry 3.
<u>2121</u>	$4 \times 3 + 3 = 30$, that is, $3 \times 5 + 0$. Show how
24338	the rest of the work is done.

EXAMPLE 2. Multiply 3 yd., 2 ft., 9 in. by 5.

$$\begin{array}{r} 3 \text{ yd.} \quad 2 \text{ ft.} \quad 9 \text{ in.} \\ \quad \quad \quad 5 \\ \hline 19 \text{ yd.} \quad 1 \text{ ft.} \quad 9 \text{ in.} \end{array}$$

EXPLANATION. $5 \times 9 \text{ in.} = 45 \text{ in.} = 3 \text{ ft.} 9 \text{ in.}$ Write 9 and carry 3. $5 \times 2 \text{ ft.} + 3 \text{ ft.} = 13 \text{ ft.} = 4 \text{ yd.} 1 \text{ ft.}$ Write the 1 ft. and carry 4 yd. $5 \times 3 \text{ yd.} + 4 \text{ yd.} = 19 \text{ yd.}$

EXAMPLE 3. Multiply 36 by $\frac{1}{5}$.

This means that we are to find $\frac{1}{5}$ of 36, which is 4, then to multiply 4 by 5. Hence $\frac{1}{5} \times 36 = 20$.

EXAMPLE 4. Find $\frac{3}{5}$ of $\frac{5}{8}$; that is, multiply $\frac{5}{8}$ by $\frac{3}{5}$.

This means that we are to find $\frac{1}{5}$ of $\frac{5}{8}$, then take 3 times the result. Since we cannot take $\frac{1}{5}$ of the 5 things, we take a unit $\frac{1}{5}$ as large, and leave the number of units the same. Then $\frac{1}{5}$ of $\frac{5}{8} = \frac{1}{8}$. If we take 3 of them, we have $\frac{3}{8}$. Hence $\frac{3}{5} \times \frac{5}{8} = \frac{3}{8}$.

Thus it is seen that

I. *A fractional multiplier denotes a double operation, the denominator being a divisor and the numerator a multiplier. And,*

II. *The product of two or more fractions is the product of the numerators for the numerator and the product of the denominators for the denominator.*

EXAMPLE 5. Find $.28 \times 345.65$

$$\begin{array}{r} 345.65 \\ .28 \\ \hline 276520 \\ 69130 \\ \hline 96.7820 \end{array}$$

EXPLANATION. Since to multiply by a fraction is to divide by the denominator and multiply by the numerator, we divide by 100, the denominator expressed by the decimal point, and multiply by 28.

But since our scale of notation is 10, to multiply by 100 is to move the decimal point two places to the left. Hence, *we multiply as in integers and point off as many decimal places as there are in both factors.*

DRILL EXERCISES.

Find the products:

1. $64 \times 38\frac{3}{4}$

3. $3475 \times 173\frac{11}{25}$

5. $341 \times 17\frac{3}{11}$

2. $396 \times 47\frac{5}{6}$

4. $1696 \times 148\frac{5}{18}$

6. $927 \times 84\frac{1}{6}$

7. Find $\frac{10}{12} \times \frac{24}{35}$.

$$\frac{10}{12} \times \frac{24}{35} = \frac{10 \times 24}{12 \times 35} = \frac{240}{420} = \frac{4}{7}$$

(a) Of what use is cancellation?

(b) On what principle is it based?

(c) Which is easier, to change the product to its lowest terms or to cancel first?

$$\text{Or, } \frac{1\cancel{0}}{1\cancel{2}} \times \frac{\cancel{2}4}{\cancel{3}5} = \frac{4}{7}$$

8. $\frac{3}{10} \times \frac{2}{7}$.

11. $\frac{25}{88} \times \frac{24}{70}$.

14. $\frac{11}{12} \times \frac{96}{121}$.

9. $\frac{4}{17} \times \frac{51}{80}$.

12. $\frac{3}{22} \times \frac{55}{88}$.

15. $\frac{35}{4} \times \frac{72}{95}$.

10. $\frac{13}{21} \times \frac{35}{88}$.

13. $\frac{5}{13} \times \frac{65}{41}$.

16. $\frac{85}{98} \times \frac{42}{54}$.

17. $8\frac{3}{4} \times 6\frac{2}{7} = \frac{35}{4} \times \frac{45}{7} = \text{what?}$

18. $\frac{2}{3} \times 7\frac{1}{2}$.

21. $6\frac{5}{8} \times 246$.

24. $\frac{3}{4} \times 1\frac{2}{7} \times \frac{7}{5}$.

19. $2\frac{3}{8} \times 7\frac{5}{16}$.

22. $\frac{4}{7} \times \frac{3}{8} \times 3\frac{1}{2}$.

25. $1\frac{2}{3} \times 5\frac{1}{4} \times 1\frac{3}{8} \times \frac{1}{5}$.

20. $4\frac{1}{2} \times 15\frac{5}{8}$.

23. $2\frac{2}{3} \times 4\frac{1}{5} \times 7\frac{1}{2} \times \frac{1}{3}$.

26. $1\frac{3}{8} \times \frac{7}{11} \times 3\frac{1}{6}$.

Multiply the following by 10; by 100; by 1000. Give the results at sight :

27. 345.68.

30. 1754.6.

33. 86.54.

28. 197.34.

31. 18.462.

34. .342.

29. 84.965.

32. 396.62.

35. 1.568.

36. How would you multiply 324.63 by 8? By 64? By 512?

37. How would you multiply 349e₁₂ by 12? By 144?

Find the products of :

38. .84 × .251.

42. 1.06 × .934.

46. 3.04 × 1.09.

39. .164 × .28.

43. 2.002 × .163.

47. .109 × .206.

40. .38 × .64.

44. 2.5 × .016.

48. .019 × .008.

41. .08 × .008.

45. 4.3 × .009.

49. .909 × 9.09.

31. Cross multiplication. When the multiplier contains but two figures, the work can easily be kept in mind and the

partial products added without writing them down. The method is best shown by examples.

EXAMPLE 1. Find the product of 48×36 .

48		EXPLANATION. $6 \times 8 = 48$. Write 8, carry 4.
36		$6 \times 4 + 8 \times 8 + 4$ (carried) = 52. Write 2, carry 5.
<u>1728</u>		$3 \times 4 + 5$ (carried) = 17.

EXAMPLE 2. Find the product of 342×86 .

342		EXPLANATION. $6 \times 2 = 12$. Write 2, carry 1.
86		$6 \times 4 + 8 \times 2 + 1$ (carried) = 41. Write 1, carry 4.
<u>29412</u>		$6 \times 3 + 8 \times 4 + 4$ (carried) = 54. Write 4, carry 5.
		$8 \times 3 + 5$ (carried) = 29.

While this method may be used with larger numbers, the larger the numbers the greater concentration required, especially where there are more than two figures in the multiplier.

DRILL EXERCISES.

Write out the products by cross multiplication :

1.	2.	3.	4.	5.	6.	7.	8.
54	98	86	73	65	45	89	43
<u>27</u>	<u>43</u>	<u>35</u>	<u>24</u>	<u>37</u>	<u>84</u>	<u>44</u>	<u>98</u>
9.	10.	11.	12.	13.	14.	15.	16.
76	59	84	38	196	234	342	843
<u>49</u>	<u>65</u>	<u>75</u>	<u>46</u>	<u>87</u>	<u>79</u>	<u>63</u>	<u>65</u>
17. 46×23 .		22. 72×86 .		27. 24×138 .			
18. 38×65 .		23. 43×19 .		28. 26×237 .			
19. 42×96 .		24. 82×96 .		29. 54×132 .			
20. 84×27 .		25. 34×71 .		30. 63×247 .			
21. 35×64 .		26. 46×59 .		31. 73×361 .			

32. Short processes in multiplication. By observing certain facts, much of the work of cross multiplication may be saved in *special kinds* of exercises in which each factor has but two figures.

EXAMPLE 1. Find the product of 74×76 .

74	EXPLANATION. $6 \times 4 = 24$. Then $6 \times 70 + 70 \times 4$, or
76	$6 \times 70 + 4 \times 70$, is 10×70 . Next we have 70×70 . Hence,
<u>5624</u>	in all, we have $70 \times 70 + 10 \times 70 + 24 = 80 \times 70 + 24 = 5624$.

Hence, it may be seen that

When the number of tens is the same in both factors, and the sum of the ones is ten, we take the product of the number of tens by the next higher number, and annex the product of the ones.

Thus, $34 \times 36 = 1224$; $82 \times 88 = 7216$; $93 \times 97 = 9021$; $72 \times 78 = 5616$.

EXAMPLE 2. Find the product of 66×37 .

66	EXPLANATION. $7 \times 6 = 42$. Then $7 \times 60 + 3 \times 60 = 10 \times 60$. Next,
37	30×60 . Hence, in all, the products are $30 \times 60 + 10 \times 60 + 42 =$
<u>2442</u>	$40 \times 60 + 42$.

Hence it may be seen that

When both figures of the multiplicand are the same, and the sum of the numbers represented by the figures of the multiplier is ten, we annex the product of the ones to the product of the number of tens in the multiplicand by the number in the multiplier increased by one.

Thus, $44 \times 64 = 2816$; $66 \times 82 = 5412$; $33 \times 73 = 2409$; $44 \times 91 = 4004$.

NOTE. While there are other similar "short methods," the two given are the most easily seen and most easily remembered. Unless kept in constant use, an attempt to memorize many such methods would result in confusion.

DRILL EXERCISES.

Give at sight the following :

- | | | | |
|---------------------|----------------------|----------------------|----------------------|
| 1. 23×27 . | 7. 33×28 . | 13. 21×29 . | 19. 93×97 . |
| 2. 16×14 . | 8. 44×64 . | 14. 42×48 . | 20. 81×89 . |
| 3. 18×12 . | 9. 55×82 . | 15. 66×19 . | 21. 99×37 . |
| 4. 26×24 . | 10. 33×73 . | 16. 54×56 . | 22. 88×64 . |
| 5. 32×38 . | 11. 44×19 . | 17. 77×28 . | 23. 84×86 . |
| 6. 46×44 . | 12. 55×28 . | 18. 88×46 . | 24. 94×96 . |

25. Why is the square of a number ending in 5 a special case of Example 1?

26. Square the following:

25; 45; 35; 65; 85; 55; 95; 75; 105; 115.

33. Multiplying by the method of aliquot parts. An aliquot part of a number is one of the integral number of parts of the number.

Thus, 6 is an aliquot part of 12; that is, it is contained in twelve an integral number of times. 25 is an aliquot part of 100; $3\frac{1}{4}$ of 10; 4 of 16; etc.

Certain aliquot parts of 10, 100, 1000, etc., occur so often that they should be recognized instantly.

Give the following:

$\frac{1}{2}$ of 10	$\frac{1}{2}$ of 100	$\frac{1}{4}$ of 100	$\frac{1}{4}$ of 100
$\frac{1}{3}$ of 10	$\frac{1}{4}$ of 100	$\frac{1}{8}$ of 100	$\frac{1}{8}$ of 100
$\frac{1}{4}$ of 10	$\frac{1}{8}$ of 100	$\frac{1}{16}$ of 100	$\frac{1}{16}$ of 1000.

MEMORIZE THIS TABLE.

$25 = \frac{1}{4}$ of 100.	$16\frac{2}{3} = \frac{1}{6}$ of 100.	$6\frac{1}{4} = \frac{1}{16}$ of 100.	$2\frac{1}{2} = \frac{1}{4}$ of 10.
$33\frac{1}{3} = \frac{1}{3}$ of 100.	$12\frac{1}{2} = \frac{1}{8}$ of 100.	$3\frac{1}{3} = \frac{1}{3}$ of 10.	$125 = \frac{1}{8}$ of 1000.

EXAMPLE 1. Multiply 42 by $16\frac{2}{3}$.

$$\begin{array}{r} 6)4200 \\ \hline 700 \end{array}$$

EXPLANATION. Since $16\frac{2}{3}$ is $\frac{1}{6}$ of 100, we may multiply by 100 and divide by 6.

DRILL EXERCISES.

1. Observe the above table, and tell how to multiply by 25. By $83\frac{1}{3}$.

2. How would you multiply by $12\frac{1}{2}$? By 125?

3. How would you multiply by $6\frac{1}{4}$? By $3\frac{1}{3}$? By $2\frac{1}{2}$?

Give at sight:

4. $83\frac{1}{3} \times 18$.

6. $16\frac{2}{3} \times 36$.

8. 25×84 .

5. 25×24 .

7. $33\frac{1}{3} \times 42$.

9. $12\frac{1}{2} \times 24$.

- | | | |
|--------------------------------|--------------------------------|--------------------------------|
| 10. $16\frac{2}{3} \times 54.$ | 13. $12\frac{1}{2} \times 72.$ | 16. $16\frac{2}{3} \times 24.$ |
| 11. $25 \times 96.$ | 14. $25 \times 32.$ | 17. $12\frac{1}{2} \times 96.$ |
| 12. $6\frac{1}{4} \times 48.$ | 15. $33\frac{1}{3} \times 68.$ | 18. $6\frac{1}{4} \times 64.$ |

Find the following products :

- | | | |
|------------------------------------|------------------------------------|------------------------------------|
| 19. $8\frac{1}{3} \times 68.72.$ | 27. $25 \times 9.368.$ | 35. $33\frac{1}{3} \times 46.751.$ |
| 20. $2\frac{1}{2} \times 86.48.$ | 28. $25 \times 73.46.$ | 36. $16\frac{2}{3} \times 32.968.$ |
| 21. $16\frac{2}{3} \times 198.52.$ | 29. $12\frac{1}{2} \times 6.344.$ | 37. $12\frac{1}{2} \times 175.82.$ |
| 22. $16\frac{2}{3} \times 884.54.$ | 30. $12\frac{1}{2} \times 73.824.$ | 38. $12\frac{1}{2} \times 134.56.$ |
| 23. $25 \times 36.978.$ | 31. $6\frac{1}{4} \times 86.375.$ | 39. $25 \times 643.96.$ |
| 24. $25 \times 32.847.$ | 32. $16\frac{2}{3} \times 42.963.$ | 40. $125 \times 34.69.$ |
| 25. $83\frac{1}{3} \times 96.126.$ | 33. $83\frac{1}{3} \times 19.634.$ | 41. $125 \times 28.34.$ |
| 26. $83\frac{1}{3} \times 73.245.$ | 34. $25 \times 263.45.$ | 42. $125 \times 36.82.$ |

34. Special fractions. There are certain decimal and common fractions which occur so frequently that one ought to be able to change instantly from one form to the other.

A TABLE OF SIMPLE RELATIONS.

$.50 = \frac{1}{2}.$	$.66\frac{2}{3} = \frac{2}{3}.$	$.62\frac{1}{2} = \frac{5}{8}.$
$.25 = \frac{1}{4}.$	$.33\frac{1}{3} = \frac{1}{3}.$	$.87\frac{1}{2} = \frac{7}{8}.$
$.12\frac{1}{2} = \frac{1}{8}.$	$.16\frac{2}{3} = \frac{1}{6}.$	$.14\frac{2}{7} = \frac{1}{7}.$
$.06\frac{1}{4} = \frac{1}{16}.$	$.08\frac{1}{3} = \frac{1}{12}.$	$.11\frac{1}{3} = \frac{1}{9}.$
$1.25 = 1\frac{1}{4}.$	$.37\frac{1}{2} = \frac{3}{8}.$	$.83\frac{1}{3} = \frac{5}{6}.$

There are other relations that are readily seen from these.

Thus, $.28\frac{1}{4} = \frac{23}{80}; .56\frac{1}{4} = \frac{23}{40}; .58\frac{1}{4} = \frac{1}{16};$ etc.

DRILL EXERCISES.

Give at sight:

- | | | |
|----------------------|----------------------|----------------------|
| 1. $28 \times .25.$ | 4. $96 \times .75.$ | 7. $24 \times 2.50.$ |
| 2. $.75 \times 84.$ | 5. $12 \times 2.25.$ | 8. $36 \times 1.25.$ |
| 3. $32 \times 1.25.$ | 6. $16 \times 1.75.$ | 9. $14 \times 2.50.$ |

- | | | |
|----------------------------------|-----------------------------------|----------------------------------|
| 10. $3.5 \times 42.$ | 23. $96 \times .08\frac{1}{2}.$ | 36. $80 \times .62\frac{1}{2}$ |
| 11. $1.25 \times 64.$ | 24. $48 \times 1.08\frac{1}{3}.$ | 37. $56 \times .87\frac{1}{2}.$ |
| 12. $24 \times .33\frac{1}{3}.$ | 25. $24 \times .37\frac{1}{2}.$ | 38. $16 \times 2.87\frac{1}{2}.$ |
| 13. $36 \times 1.33\frac{1}{3}.$ | 26. $16 \times 1.37\frac{1}{2}.$ | 39. $32 \times 1.87\frac{1}{2}.$ |
| 14. $18 \times .66\frac{2}{3}.$ | 27. $.83\frac{1}{3} \times 36.$ | 40. $40 \times 1.37\frac{1}{2}.$ |
| 15. $48 \times 1.66\frac{2}{3}.$ | 28. $24 \times .83\frac{1}{3}.$ | 41. $48 \times .87\frac{1}{2}.$ |
| 16. $51 \times .33\frac{1}{3}.$ | 29. $18 \times 1.83\frac{1}{3}.$ | 42. $35 \times 1.14\frac{2}{7}.$ |
| 17. $12 \times 3.50.$ | 30. $126 \times 1.33\frac{1}{3}.$ | 43. $42 \times 2.33\frac{1}{3}.$ |
| 18. $54 \times .16\frac{2}{3}.$ | 31. $480 \times 1.66\frac{2}{3}.$ | 44. $48 \times 1.87\frac{1}{2}.$ |
| 19. $24 \times 1.16\frac{2}{3}.$ | 32. $49 \times .14\frac{2}{7}.$ | 45. $72 \times 1.11\frac{1}{9}.$ |
| 20. $.12\frac{1}{2} \times 88.$ | 33. $12 \times .12\frac{1}{2}.$ | 46. $36 \times .44\frac{4}{9}.$ |
| 21. $1.12\frac{1}{2} \times 24.$ | 34. $24 \times .37\frac{1}{2}.$ | 47. $27 \times 1.22\frac{2}{9}.$ |
| 22. $72 \times .16\frac{2}{3}.$ | 35. $48 \times .62\frac{1}{2}.$ | 48. $56 \times 1.62\frac{1}{2}.$ |

35. The order of factors. The product of several factors is the same, whatever the order of the factors. There is, however, often a choice in the order.

Thus, $17 \times 25 \times 4$ is plainly 17×100 or 1700 , which is much easier than to find 17×25 , then 425×4 .

DRILL EXERCISES.

Give at sight :

- | | | |
|-----------------------------|---|---|
| 1. $13 \times 4 \times 5.$ | 7. $16\frac{2}{3} \times 45 \times 6.$ | 13. $76 \times 5 \times 20.$ |
| 2. $4 \times 18 \times 5.$ | 8. $93 \times 12\frac{1}{2} \times 8.$ | 14. $2\frac{1}{2} \times 16 \times 40.$ |
| 3. $15 \times 25 \times 4.$ | 9. $8 \times 46 \times 12\frac{1}{2}.$ | 15. $33\frac{1}{3} \times 173.$ |
| 4. $17 \times 8 \times 5.$ | 10. $3 \times 17 \times 33\frac{1}{3}.$ | 16. $24 \times 25 \times 4.$ |
| 5. $20 \times 63 \times 5.$ | 11. $6\frac{1}{4} \times 28 \times 16.$ | 17. $18 \times 5 \times 17.$ |
| 6. $25 \times 24 \times 2.$ | 12. $35 \times 6 \times 16\frac{2}{3}.$ | 18. $26 \times 4 \times 25.$ |

Apply any of the methods you have had and give results at sight :

- | | | |
|----------------------|----------------------|---------------------------------|
| 19. 73×77 . | 28. 24×53 . | 37. $16\frac{2}{3} \times 48$. |
| 20. 91×99 . | 29. 32×46 . | 38. $12\frac{1}{2} \times 72$. |
| 21. 88×82 . | 30. 31×76 . | 39. 25×28 . |
| 22. 77×64 . | 31. 42×69 . | 40. 25×64 . |
| 23. 65×65 . | 32. 46×81 . | 41. $16\frac{2}{3} \times 54$. |
| 24. 85×85 . | 33. 46×93 . | 42. $12\frac{1}{2} \times 48$. |
| 25. 87×83 . | 34. 32×97 . | 43. $12\frac{1}{2} \times 24$. |
| 26. 96×94 . | 35. 48×24 . | 44. $6\frac{1}{4} \times 48$. |
| 27. 31×39 . | 36. 67×63 . | 45. $6\frac{1}{4} \times 64$. |

At sight, give the cost of :

- | | |
|---|---|
| 46. 21 lb. coffee at $33\frac{1}{3}$ ct. | 54. 96 lb. butter at 25 ct. |
| 47. 48 lb. tea at 25 ct. | 55. 48 qt. berries at $12\frac{1}{2}$ ct. |
| 48. 24 lb. prunes at $16\frac{2}{3}$ ct. | 56. 9 yd. cloth at 50 ct. |
| 49. 32 lb. rice at $6\frac{1}{4}$ ct. | 57. 16 yd. cloth at 25 ct. |
| 50. 48 lb. raisins at $16\frac{2}{3}$ ct. | 58. 12 yd. lace at $8\frac{1}{3}$ ct. |
| 51. 48 lb. steak at $12\frac{1}{2}$ ct. | 59. 36 yd. braid at $8\frac{1}{3}$ ct. |
| 52. 54 lb. steak at $16\frac{2}{3}$ ct. | 60. 24 yd. cloth at $16\frac{2}{3}$ ct. |
| 53. 72 lb. lard at $16\frac{2}{3}$ ct. | 61. 96 yd. cloth at $12\frac{1}{2}$ ct. |

36. Other methods of saving work. If the figure 1 should occur in the multiplier, especially as the first or last figure, the multiplicand may be used for the first partial product.

EXAMPLE 1. Multiply 385 by 81.

WORK. EXPLANATION.

$$\begin{array}{r} 385 = 1 \times 385 \\ 3080 = 80 \times 385 \\ \hline 31185 = 81 \times 385 \end{array}$$

EXAMPLE 2. Multiply 463 by 17.

WORK. EXPLANATION.

$$\begin{array}{r} 463 = 10 \times 463 \text{ (why ?)} \\ 3241 = 7 \times 463 \\ \hline 7871 = 17 \times 463 \end{array}$$

37. The sum or difference of products having a common factor. By observing such combinations as $4 \times 7 + 6 \times 7 = 10 \times 7$, or 70, time may be saved.

EXAMPLE 1. Find the value of $17 \times 48 + 83 \times 48$.

$$17 \times 48 + 83 \times 48 = 100 \times 48, \text{ or } 4800.$$

EXAMPLE 2. Find the value of $45 \times 64 + 38 \times 64 - 33 \times 64$.

$$45 \times 64 + 38 \times 64 - 33 \times 64 = 50 \times 64 = \frac{1}{4} \text{ of } 6400 = 8200.$$

DRILL EXERCISES.

Find the value of :

1. 731×345 .

6. $38 \times 346 + 42 \times 346$.

2. 126×489 .

7. $64 \times 575 + 85 \times 575 - 24 \times 575$.

3. 231×967 .

8. $9 \times 3.1416 + 16 \times 3.1416$.

4. 146×3.45 .

9. $2 \times 16 \times 8 + 2 \times 24 \times 8 + 16 \times 24$

5. 3.41×79.6 .

10. $365 \times 784 - 15 \times 784 + 50 \times 784$.

CHAPTER V.

DIVISION, FACTORS, ETC.

38. Division Division, like subtraction, is an *inverse* process. In division we have given the product of two factors, and one of the factors, and are to find the other. Then, *division is the inverse of multiplication.*

39. Two kinds of division. Since one factor of a product may be concrete, in which case the product is like the concrete factor, we may have a concrete number (a *dividend*) to be divided by either an abstract or concrete number (the *divisor*).

Thus, since $3 \times \$4 = \12 , (1) $\$12 \div \$4 = 3$, and (2) $\$12 \div 3 = \4 .

(a) *When the dividend and divisor are alike, the quotient shows how many times the dividend contains the divisor.*

Just as the multiplier shows how many times the multiplicand must be used as an *addend* to produce the product, the quotient in this kind of division shows how many times the divisor may be *subtracted* from the dividend.

By some this kind of division is called *comparison*, and by others, *measurement*, for it really measures the dividend by using the divisor as a unit of measure. This kind of division must be expressed by the sign of division, as $\$18 \div \3 , usually read “\$18 will contain \$3.”

(b) *When the divisor is abstract, it shows into how many parts the dividend is to be divided, and the quotient shows the size of the parts.*

This kind of division, then, consists of dividing a number into equal parts, and hence is called by some *partition*, or *partitive division*. This kind of division may be expressed by the sign of division, or by use of a fraction and the word “of.” Thus, to express the fact that if \$36 is divided into 4

equal parts, each part will be \$9, we may write $\$36 \div 4 = \9 , or $\frac{1}{4}$ of $\$36 = \9 . The latter is perhaps most used.

NOTE.—It will be observed that $\$36 \div 4$ is not to be read “ $\$36$ will contain 4.” The sign is read “divided by” in such cases.

40. The process of division. Divisor integral. In explaining the process of division, either of the above interpretations may be used. Since the number of units represented by the quotient is independent of these interpretations, we shall consider the divisor abstract.

EXAMPLE 1. Divide 3248 by 7.

$$\begin{array}{r} 7 \overline{) 3248} \\ \underline{464} \end{array}$$

EXPLANATION. Dividing any number into parts does not change the unit. Hence $\frac{1}{7}$ of 32 *hundreds* = 4 *hundreds* and 4 *hundreds* yet undivided. 4 *hundreds* and 4 *tens* = 44 *tens*. $\frac{1}{7}$ of 44 *tens* = 6 *tens* and 2 *tens* yet undivided. 2 *tens* and 8 = 28. $\frac{1}{7}$ of 28 = 4.

EXAMPLE 2. Divide 35587 by 56.

$$\begin{array}{r} 635 \\ 56 \overline{) 35587} \\ \underline{336} \\ 198 \\ \underline{168} \\ 307 \\ \underline{280} \\ 27, \text{ remainder} \end{array}$$

EXPLANATION. The first left hand figures which represent a number that will contain 56 are 355. We do not need even to notice the unit (in this case hundreds) represented by it. $\frac{1}{56}$ of 355 of any unit = 6 of that unit and 19 remaining undivided. The 6 is placed above the right hand figure of the number divided to show that it is of the same unit as the number divided. To “bring down” 8 is equivalent to multiplying the 19 by 10 and adding 8.

That is, by bringing down the next figure of the dividend, we are really changing the number remaining after each division to the unit of the next lower order and adding it to the number of that order.

EXAMPLE 3. Divide .8064 by 64.

$$\begin{array}{r} .0126 \\ 64 \overline{) .8064} \\ \underline{64} \\ 166 \\ \underline{128} \\ 384 \\ \underline{384} \end{array}$$

Explain why the first quotient figure is written in *hundredths'* place. (See the explanation in Example 2.)

EXAMPLE 4. Divide 45 ft. 4 in. into 8 equal parts.

$$\begin{array}{r} \text{E)} 45 \text{ ft. } 4 \text{ in.} \\ \underline{5 \text{ ft. } 8 \text{ in.}} \end{array}$$

EXPLANATION. $\frac{1}{8}$ of 45 ft. = 5 ft. and 5 ft. yet undivided. 5 ft. = 60 in. 60 in. + 4 in. = 64 in. $\frac{1}{8}$ of 64 in. = 8 in.

EXERCISES.

1. Give examples to illustrate each kind of division.
2. Which kind is $\$85 \div \17 ? Subtract $\$17$ from $\$85$, then from the remainder, and so on, until no remainder is left. How many times do you subtract $\$17$?
3. Which kind of division was used to explain the process in the illustrative examples above? Use the other kind, and explain the process.

Divide:

- | | |
|-------------------|----------------------------------|
| 4. 35.265 by 84. | 12. 34 ft. 6 in. by 8. |
| 5. 396.84 by 19. | 13. 28 lb. 12 oz. by 9. |
| 6. 63.245 by 35. | 14. $84^\circ 15' 38''$ by 15. |
| 7. 9.3456 by 185. | 15. $78^\circ 23' 42''$ by 15. |
| 8. 10.463 by 89. | 16. 35 bu. 3 pk. 2 qt. by 8. |
| 9. 1.3468 by 73. | 17. 16 gal. 2 qt. 1 pt. by 5. |
| 10. 346.28 by 82. | 18. 16 yd. 2 ft. 8 in. by 9. |
| 11. 75.642 by 64. | 19. 4 hr. 32 min. 20 sec. by 12. |

41. Division of numbers in any scale. It must have been observed that the process of division is independent of the decimal scale of relation. It will be interesting to perform some divisions of numbers represented in other scales.

EXAMPLE 1. Divide 42321, by 134.

$$\begin{array}{r} 224 \\ 134 \overline{) 42321} \\ \underline{828} \\ 1002 \\ \underline{828} \\ 1241 \\ \underline{1201} \\ 40 \end{array}$$

Show how each figure in the work is obtained.

EXAMPLE 2. Change $\frac{5}{6}$ to a place-value fraction whose scale is 5.

$$\begin{array}{r} 5 \underline{= 10}_6 \\ 6 \underline{= 11}_6 \end{array} \quad \begin{array}{r} .404 + \\ 11 \overline{) 10.000} \\ \underline{4.4} \\ 100 \\ \underline{44} \\ 1 \end{array}$$

EXPLANATION. Expressing 5 and 6 in the scale of 5, we have 10 and 11. Now dividing as in Example 1, we have .404....

DRILL EXERCISES.

Divide :

- | | | |
|---|--|--|
| 1. 3242 ₅ by 41 ₅ . | 5. 20101 ₅ by 121 ₅ . | 9. 34253 ₅ by 512 ₅ . |
| 2. 13042 ₅ by 32 ₅ . | 6. 102201 ₅ by 211 ₅ . | 10. 53241 ₅ by 155 ₅ . |
| 3. 36245 ₅ by 173 ₅ . | 7. 83te ₁₁ by 1e5 ₁₁ . | 11. et05e ₁₁ by 1e4 ₁₁ . |
| 4. 73426 ₅ by 137 ₅ . | 8. e5t6 ₁₁ by 62t ₁₁ . | 12. 101101 ₅ by 111 ₅ . |

Change to place value fractions, (sometimes called *radix* fractions) :

- | | | |
|---------------------------------|----------------------------------|---------------------------------|
| 13. $\frac{3}{5}$, scale of 5. | 15. $\frac{7}{5}$, scale of 12. | 17. $\frac{3}{5}$, scale of 6. |
| 14. $\frac{5}{9}$, scale of 5. | 16. $\frac{3}{5}$, scale of 8. | 18. $\frac{7}{9}$, scale of 4. |

42. Dividing by a fraction. When the divisor is a fraction, the problem may be changed to one similar to those of the preceding sections by multiplying both dividend and divisor by the same number. For it may be seen that:

Multiplying both dividend and divisor (or both terms of a fraction) by the same number does not change the value of the quotient (or fraction).

EXAMPLE 1. Divide 84.24 by .8.

$$\begin{array}{r} .8 \overline{) 84.24} \\ 8 \overline{) 842.4} \\ \underline{42.8} \end{array}$$

Observe that to make an integer of the divisor both divisor and dividend are multiplied by 10, i. e., the decimal point is moved one place to the right in both the divisor and the dividend.

EXAMPLE 2. Divide $\frac{5}{3}$ by $\frac{2}{3}$.

$$\frac{5}{3} \div \frac{2}{3} = 20 \div 21, \text{ or } \frac{20}{21}$$

EXPLANATION. Here we multiply both fractions by 28, the L. C. M. of the denominators.

Observe that the 5 was multiplied by 4 and the 3 by 7. This, it will be observed, is *equivalent to multiplying the dividend by the divisor inverted*.

EXAMPLE 3. Divide $\frac{3}{5}$ by $\frac{4}{7}$.

$$\begin{array}{r} 3 \\ 5 \times \frac{7}{4} = \frac{9}{20} \\ 4 \end{array}$$

EXPLANATION. The dividend is multiplied by the divisor inverted. In general this method is shorter than that of Example 2.

DRILL EXERCISES.

By inspection give the quotients :

- | | | | |
|---------------------|----------------------|----------------------|-----------------------|
| 1. $1 \div .1$. | 6. $1 \div 10$. | 11. $2 \div .1$. | 16. $.02 \div 2$. |
| 2. $1 \div .01$. | 7. $1 \div 100$. | 12. $2 \div .01$. | 17. $.02 \div .001$. |
| 3. $10 \div .1$. | 8. $1 \div .0001$. | 13. $.2 \div .02$. | 18. $3 \div .1$. |
| 4. $10 \div .01$. | 9. $10 \div 100$. | 14. $.02 \div 2$. | 19. $3 \div .01$. |
| 5. $10 \div .001$. | 10. $10 \div 1000$. | 15. $.2 \div .002$. | 20. $3 \div .001$. |

Find the quotients :

- | | | |
|---------------------------------------|--|--|
| 21. $84.62 \div 32.4$. | 25. $9.63 \div .25$. | 29. $.973 \div 2.61$. |
| 22. $7.68 \div 2.4$. | 26. $10.6 \div .72$. | 30. $.8 \div .73$. |
| 23. $89.6 \div .42$. | 27. $12.8 \div 4.2$. | 31. $.95 \div .012$. |
| 24. $.782 \div .032$. | 28. $.873 \div 1.46$. | 32. $.84 \div .009$. |
| 33. $\frac{5}{7} \div \frac{3}{8}$. | 36. $\frac{9}{7} \div \frac{11}{12}$. | 39. $\frac{3}{4} \div \frac{5}{6}$. |
| 34. $\frac{6}{11} \div \frac{4}{9}$. | 37. $\frac{4}{3} \div \frac{7}{12}$. | 42. $\frac{5}{44} \div \frac{2}{11}$. |
| 35. $\frac{6}{7} \div \frac{3}{14}$. | 40. $\frac{3}{4} \div \frac{5}{16}$. | 43. $.32 \div \frac{1}{7}$. |
| 38. $\frac{4}{5} \div \frac{7}{15}$. | 41. $\frac{2}{27} \div \frac{5}{10}$. | 44. $.042 \div \frac{7}{25}$. |

43. Precedence of signs. It may happen that in expressing all of the work to be done in solving a problem, several of the processes occur. When the signs of addition and subtraction alone, or the signs of multiplication and division alone, occur, the processes are performed in order from left to right.

Thus, in $12 - 6 - 2 + 8$, $12 - 6 = 6$; $6 - 2 = 4$; $4 + 8 = 12$. In $6 \times 20 + 8 \times 3$, $6 \times 20 = 120$; $120 + 8 = 128$; $15 \times 3 = 45$.

When the signs of all four processes occur, mathematicians have agreed that

The processes of multiplication and division must be performed before those of addition and subtraction.

Thus, $3 \times 7 - 6 \times 2 + 15 \div 3 = 21 - 12 + 5 = 14$.

44. Factors. On account of the decimal place-value feature of our notation, certain factors of numbers consisting of several digits may easily be seen from the sum of the digits, or from the nature of the last digits, etc.

I. *The factors 2 and 5.* Since 2 and 5 are factors of 10, they are evidently factors of any multiple of 10. Also, since any number may be considered a number of 10's plus the last digit of the number, it follows that

Any number will contain 2 or 5 if the last digit will, and not otherwise.

II. *The factors 4 and 25.* Since 100 will contain both 4 and 25, any multiple of 100 will contain both 4 and 25. Also, since any number may be considered as a number of 100's plus the number represented by the last two digits, it follows that

Any number will contain 4 or 25 if the number represented by the last two digits will, and not otherwise.

III. *The factors 9 and 11.* It may easily be seen that by taking 1 from any power of 10, we have a number whose digits are each 9 and which, therefore, will contain 9. Also by taking 1 from any even power of 10, or by adding 1 to any odd power of 10, we have a number that will contain 11. It is also known that any number in the scale of 10 consists of the sum of multiples of powers of 10. From these two facts we can easily find tests for divisibility by 9 and 11.

(a) *Divisibility by 9.* Take the number 34267. It is equal

to $3 \times 10000 + 4 \times 1000 + 2 \times 100 + 6 \times 10 + 7$. This is also equal to $3 \times 9999 + 3 + 4 \times 999 + 4 + 2 \times 99 + 2 + 6 \times 9 + 6 + 7$, from which we see that the number will contain 9 if the sum of the digits $3 + 4 + 2 + 6 + 7$ will contain 9. Since any number may be separated in this way, it is seen that

Any number will contain 9 if the sum of the digits will contain 9, and not otherwise.

(b) *Divisibility by 11.* The number above is also evidently equal to $3 \times 9999 + 3 + 4 \times 1001 - 4 + 2 \times 99 + 2 + 6 \times 11 - 6 + 7$, from which we see that the number will contain 11 if the difference between the sums of the odd and even placed digits will contain 11. And, in general,

Any number will contain 11 if the difference between the sums of the odd and even placed digits will contain 11, and not otherwise.

EXERCISES.

1. Extend the reasoning of § 44 to tests of divisibility by 8, 16, 125, and 625.

2. Show why the divisibility by any factors except those made up of 2 or 5, or powers of these numbers, or their products, cannot depend upon the number represented by any part of the given number.

3. Had 5 been the scale of our system of notation, would the divisibility by any factor depend upon any particular digit or last digits? Why?

4. With a scale of 12, what factors would have depended upon the last digit? Why?

5. With a scale of 12, upon what will divisibility by 8 depend? By 16 (sixteen)? By 9?

6. Had 8 been the scale, what factors would have depended upon the last digit? What upon the number represented by the last two digits?

7. In a scale of 12, show that divisibility by 11 depends upon the sum of the digits.

8. If we used a scale of 8, upon what would the divisibility by 7 depend?

9. With a scale of 8 show that divisibility by 9 depends upon the difference between the sums of the odd and even placed digits.

10. By properly separating any number into parts, show that a number is divisible by 3 if the sum of the digits is divisible by 3, and not otherwise.

45. Uses of arithmetic. A knowledge of numbers is indispensable in carrying on the practical affairs of life. The power to see the quantitative relations of the magnitudes about us stands next in value to the power to communicate with each other.

46. The solution of problems. The ability to compute in no way insures the power to solve problems. Through constant drill and practice, one comes to compute without giving much thought to the work. Computation requires merely very close attention and perfect familiarity with a few combinations. The ability to solve problems, however, does not become so mechanical. Power to solve problems depends upon power to see relations between magnitudes, and not upon a knowledge of fixed formal rules.

47. Problems and drill exercises. After discovering the solution of a certain class of problems, others of this class do not require the same thought and thus they become mere drill exercises. In many of the problems that arise in business, in science, or in the industries, the processes to be performed are perfectly apparent as soon as the meanings of the terms

involved are understood. These then, are but drill exercises, but those of frequent occurrence in the world's work are important. They give practice in important applications, and should be drilled upon by the student until great efficiency is attained. Parts II, III, and IV of this book show some of the most important applications of arithmetic.

48. Method of attack. To successfully attack a problem requires (1) *Power to understand the statement of the problem*; (2) *Power to observe the properties of the magnitudes and to see the relations of the magnitudes involved*; (3) *Power to determine what to do*.

The steps, then, in solving any problem are as follows:

I. *Getting a complete understanding of just what is wanted and what is given in the problem, or what is known in some other way that will help to find this.*

II. *A comparison of what is given with what is wanted in order to discover just what work is to be done, i. e., what operations are to be performed.*

III. *The processes indicated and the work accurately performed.*

IV. *A comparison of the result with the data given, to see if the result seems reasonable.*

EXAMPLE 1. A milkman added 36 cows to his herd. This was 45% of the number in the resulting herd. How many had he at first?

I. We must first know what this means. We must know that 45% of anything means .45 of the thing. We must see that the 36 cows constituted 45% of the present herd and that the remainder, or 55% of the herd, is required.

II. Now to decide what to do, we must compare what is wanted with what is given. We thus see that the number required is $\frac{1}{11}$ of the number given (comparing 55% with 45%). Hence, we know that we are to find $\frac{1}{11}$ of 36.

III. We now indicate the work, and solve it in the shortest possible way.

$$\frac{11}{9} \times 3\frac{4}{9} \text{ cows} = 44 \text{ cows.}$$

IV. Does the result seem reasonable?

EXAMPLE 2. If 50 acres of corn produce 2800 bushels, what should I expect from 75 acres producing the same yield per acre?

I. *Given* the yield of 50 acres; *required* the yield of 75 acres.

II. Comparing 75 with 50, we see that we should multiply 2800 by $1\frac{1}{2}$, or add $\frac{1}{2}$ of 2800 to 2800.

III. Work:

$$\begin{array}{r} 2800 \\ 1400 \\ \hline 4200 \end{array}$$

IV. Does the result seem reasonable?

EXAMPLE 3. A rectangular field of 54 acres is 30 rd. wide. What will it cost to fence it at $2\frac{1}{2}$ cents a foot?

I. We know that there are 160 sq. rd. in 1 acre, and that the area of a rectangle is the product of its dimensions.

II. Comparing what is wanted with what is given, we see that we must find the length, then the number of feet in the perimeter, and then multiply $2\frac{1}{2}$ cents by this.

III. Work indicated and performed:

$$\begin{array}{c} 18 \quad 16 \\ \frac{2 \times 54 \times 160 \times 33}{30} \times 2\frac{1}{2} \text{ ct.} + \frac{2 \times 30 \times 33}{2} \times 2\frac{1}{2} \text{ ct.} = 318 \times 33 \times \frac{5}{2} \text{ ct.} \end{array}$$

$$\begin{array}{c} 159 \\ 318 \times 33 \times \frac{5}{2} \text{ ct.} = \$262.35. \end{array}$$

IV. Does the result seem reasonable?

PART II

APPLICATIONS OF ARITHMETIC TO BUSINESS

CHAPTER I.

ACCOUNTS.

49. An account. A record of the *debts* and *credits* between persons having business transactions, or of the receipts and expenditures of a particular class of transactions, is called an *account*.

50. The debit side of an account. The *debts* of an account are records of *debt*. These are placed at the *left* of the central dividing line in the record of the account.

51. The credit side of an account. The *credits* of an account are records of money, or its equivalent, paid by a debtor on his account. The credits are placed on the *right* side of the central dividing line in the record of the account.

52. To balance an account. The difference between the sums of the debits and credits of an account is called the *balance*. To balance an account is to enter the balance on the lesser side, thus making the total footings of the two sides equal.

The following illustration shows the form of a ledger account between E. L. Holmes & Co., merchants, and Robt. L. Ray, debtor:

Dr.				ROBT. L. RAY				Cr.	
1908				1908					
Jan.	2	Sales	28 00	Jan.	6	Cash	20 00		
"	7	"	36 00	"	15	"	35 00		
"	10	Cash loan	10 00	Feb.	15	"	20 00		
"	15	Sales	32 50	Mar.	1	Balance	92 50		
Feb.	12	"	18 75						
"	14	"	42 25						
			167 50				167 50		

1. Who has bought goods as shown by the above account? Of whom has he bought them?

2. What do the "Sales" items of the debit side show? What does the "cash loan" item of the debit side show?

3. What do the "cash" items of the credit side show?

4. What does the "balance" on the credit side show?

5. What would a "balance" on the debit side have meant?

EXERCISES.

Balance the following accounts and tell what the balance indicates in each case:

1. John V. Farwell & Co., in account with B. L. Gray: Sales, June 1, 25 yd. brussels carpet at \$1.12; June 8, 46 yd. black silk at \$1.04; 38 yd. Wamsutta cotton at 11½ cents; June 25, 86 yd. cotton at 9¼ cents; 34 yd. silk at \$1.18. Credits, June 15, cash \$45; June 20, goods returned, \$23; June 25, cash \$20. Balance the account July 1.

2. R. C. Randall & Co., in account with W. R. Brooks: Sales, \$38.40; \$96.23; \$84.68; \$34.50; \$26.84; \$126.18; \$342.96. Credits by cash and goods returned, \$120; \$26.84; \$90; \$38.50; \$84.20.

3. Miller Bros., in account with Wm. R. West: Sales, \$84.60; \$96.80; \$64.98; \$134.18; \$28.43; \$96.84; \$38.26; \$96.24. Credits by cash and goods returned, \$100; \$34.60; \$75; \$26.80.

53. An account with Cash. When I keep an account with "Cash," I am keeping an account, as it were, with my own pocket-book or cash-box. "Cash" is *debtor*, that is, owes me, for all that is put in, and "Cash" is *credited* with all that is taken out.

Dr.		CASH		Cr.	
1908				1908	
May	1	Balance	\$100 00	May	18
"	5	Rent recd.	50 00	"	14
"	7	Mdse. sold	25 00	"	18
"	10	Land sold	725 00	"	22
			900 00		
May	22	Balance	75 00		

1. Cash is charged with having received four amounts, which it owes me, that is, for which it is my debtor. How much on hand at the beginning?

2. What is the total amount Cash has received?

3. When I take out \$350 with which to purchase a piano, Cash has paid me back how much of what it owes me?

4. What other amounts has Cash paid me, that is, for what other amounts should Cash be credited?

5. How much more has Cash received than paid out?

6. How much more might I have spent so as to balance the account?

7. For what is Cash debtor at the beginning of the next account?

EXERCISES.

1. Balance the Cash account of Charles Watson. He has on hand \$4.21. He receives at various times \$6.24, \$7.36, \$8.49, \$7.34, \$6.75. He pays out \$8.75, \$9.81, \$8.39.

2. On Monday morning a merchant begins business with \$247.84 on hand. He receives \$24.75, \$86.91, \$84.28, \$97.25, \$164.29. He pays out \$18.99, \$37.49, \$64.91, \$83.15. Find the balance on hand.

Find the balance of each of the following accounts :

3.		4.		5.	
Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
\$987.65	\$629.55	\$4768.82	\$468.34	\$649.81	\$82.46
1839.76	83.74	947.61	984.59	8439.87	981.32
6482.91	968.71	847.77	1483.22	648.38	641.25
478.85	28.46	3998.64		91.76	239.86
698.47	318.93	8372.91			728.41
6.		7.		8.	
Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
\$246.94	\$839.75	\$698.32	\$649.83	\$356.78	\$135.72
839.76	646.81	376.54	478.88	938.12	873.54
842.94	794.32	843.26	694.31	45.23	137.92
327.68	546.78	695.98	883.24	938.83	7639.85
946.32	937.89	831.96	695.64	876.23	736.29

9. The following is the open account of E. R. Walker with the First National Bank of Monroe :

Dr.		E. R. WALKER				Cr.			
Jan.	11	Check	\$500	00	Jan.	4	Balance	\$486	87
"	12	"	57	30	"	10	Deposit	290	00
"	20	"	235	75	"	11	"	198	75
"	22	"	11	80	"	21	"	773	40
"	28	"	97	30	"	25	"	110	00

Without balancing, it is evident that the balance will be on the debit side. What does this show? What would a balance on the credit side show? Find the balance.

10. Balance the following and tell what it shows :

On May 1, E. F. Mason had a balance of \$4370.28 to his account in the bank. He deposited on May 1, \$269; May 6, \$165; May 10, \$175; May 15, \$180.50; May 20, \$290. He withdrew by check the following amounts: May 1, \$156; May 10, \$450; May 13, \$125; May 27, \$675.50. What was his balance June 1?

CHAPTER II.

BUYING AND SELLING.

54. Percentage. A merchant usually thinks of his loss or gain as being a certain *per cent* of the cost. A number of per cent, as 6 per cent (written 6%), is merely a number of *hundredths*. Hence, a per cent is a fraction showing the relation of one quantity to another—a fraction whose unit is a *hundredth*.

Thus, if an article costing \$6 is sold for \$9, the gain is \$3, or $\frac{1}{2}$ of the cost. It is more common, however, to say that the gain is 50%.

Again, if an article costing \$4 is to be sold at a gain of $\frac{1}{4}$ of the cost, or for \$5, we usually say that it is sold at a gain of 25%.

55. Problems in percentage. The two problems of percentage most common in business are:

I. *To find a part of some number when the part to be found is stated in terms of per cent ;*

II. *To compare one number with another and express the relation in terms of per cent.*

Another problem very much less common in business is :

To find a number when some known part, expressed as per cent, is given.

EXAMPLE 1. A merchant bought a suit for \$12, and sold it at a gain of 30%. Find the amount gained.

SOLUTION: We are to find .30, or $\frac{3}{10}$, of \$12. $.30 \times \$12 = \3.60 , the gain.

EXAMPLE 2. A dealer gained \$3 on goods that cost him \$15. What per cent of the cost was gained ?

SOLUTION: We are to find the relation of \$3 to \$15, and express it as per cent. \$3 is $\frac{1}{5}$ of \$15. $\frac{1}{5} = 20\%$.

56. Fractional equivalents of certain per cents. There are certain per cents which are so common in business that their fractional equivalents should be recognized instantly.

A TABLE OF EQUIVALENTS.

$\frac{1}{2}$ =50%.	$\frac{1}{3}$ =33 $\frac{1}{3}$ %.	$\frac{1}{5}$ =20%.	$\frac{4}{5}$ =80%.
$\frac{1}{4}$ =25%.	$\frac{2}{3}$ =66 $\frac{2}{3}$ %.	$\frac{2}{5}$ =40%.	$\frac{3}{4}$ =75%.
$\frac{1}{6}$ =12 $\frac{1}{2}$ %.	$\frac{1}{6}$ =16 $\frac{2}{3}$ %.	$\frac{3}{5}$ =60%.	$\frac{5}{6}$ =87 $\frac{1}{2}$ %.

OTHER EQUIVALENTS LESS IMPORTANT ARE:

$\frac{5}{6}$ =83 $\frac{1}{3}$ %.	$\frac{1}{6}$ =16 $\frac{2}{3}$ %.	$\frac{1}{12}$ =8 $\frac{1}{3}$ %.
$\frac{1}{8}$ =12 $\frac{1}{2}$ %.	$\frac{1}{8}$ =12 $\frac{1}{2}$ %.	$\frac{1}{16}$ =6 $\frac{1}{4}$ %.

ORAL PROBLEMS.

1. A book that cost the dealer \$2.00 was sold at a gain of 40%. What was the gain? What was the selling price? What per cent would have been gained, had the selling price been \$2.50? What per cent would have been lost, had the selling price been \$1.75?

NOTE. The rate of gain is a per cent of the cost.

2. By selling a book for \$1.60, a merchant gained 40 cents. What per cent of gain was this? At what price should he have sold it to gain 50%? To lose 25%?

3. Which pays the better rate of gain, a suit sold for \$12 at a gain of \$3, or one sold for \$18 at a gain of \$4?

4. I lost 20% when selling an overcoat for \$15. At what price should I have sold it to gain 20%?

SUGGESTION. The known number, \$15, is 80% of the cost, and 120% of the cost is required. Compare what is wanted with what is given, and show that the solution is $1\frac{1}{2} \times \$15$, or $\$15 + \frac{1}{2}$ of \$15.

5. A merchant marked an article to sell at \$21. This was a gain of 40%. What was the per cent of gain or loss if it was sold for \$18?

SUGGESTION. At \$18, it sold for $\frac{2}{3}$ of what it was marked. That is, it sold for $\frac{2}{3}$ of 140% of the cost. Then it sold at a gain of 20%. Why?

6. A farm that cost \$6000 was sold for \$7000. What per cent was gained?

7. A house was sold for \$8000. This was \$1000 more than it cost. What was the per cent gained?

8. A stock of goods sold at \$3000. This was \$1000 less than the cost. What was the per cent of loss?

9. If I buy books at 20% below the list price, at what were books listed that cost me \$36?

SUGGESTION. 80% of the list price is given, and the whole list price or 100% of it is wanted. Show why $\frac{1}{4}$ of \$36 added to \$36 gives the result required.

10. After deducting $83\frac{1}{3}\%$ from the list price, goods cost me \$27. At what were they listed?

WRITTEN PROBLEMS.

1. A house and lot costing \$6800 is to be sold at an advance of 18% of the cost. Find the gain and the selling price.

SOLUTION.

$$\begin{array}{r} \$6800 \\ .18 \\ \hline 54400 \\ 6800 \\ \hline \$1224.00 = \text{the gain} \\ 6800 \\ \hline \$8024.00 = \text{the selling price} \end{array}$$

2. A merchant sold out his business at 75% of the invoice price of the stock of goods. If the goods were invoiced at \$6453, what did he get for them?

3. A grain dealer bought 16,500 bushels of wheat at $82\frac{1}{2}$ cents a bushel and sold it at an advance of $12\frac{1}{2}\%$. How much did he gain?

4. A farm that cost \$9600 was sold for \$8500. The loss was what per cent of the cost?

SOLUTION.

$$\begin{array}{r} \$9600 \\ 8500 \\ \hline 1100 = \text{loss} \\ \hline \text{Hence the loss} = \frac{11}{96} \text{ of the cost} \end{array}$$

$$\begin{array}{r} .11\frac{1}{16} = 11\frac{1}{16}\% \\ 96)11.00 \\ 96 \\ \hline 140 \\ 96 \\ \hline 44 = 4\frac{1}{2}\% \end{array}$$

REMARK. *A per cent is only a relation expressed as hundredths.* Hence in all such problems, we get the relation required and express it as hundredths.

5. A piano costing \$360 and selling for \$580, yields what rate of gain?

6. A traveling salesman sold \$75,500 worth of goods during a certain year. He received $7\frac{1}{2}\%$ of his sales for his services. What did he earn?

7. A salesman earned \$4250 one year when he received but $6\frac{1}{4}\%$ of his total sales. What were his total sales that year?

SUGGESTION. Since 100% of his sales, or *all* of them, are required, and $6\frac{1}{4}\%$ of them are known, the solution is $16 \times \$4250$. Why?

8. A merchant's sales for the year amounted to \$75,000. His average gain was $12\frac{1}{2}\%$ of the sales. If clerk hire was \$2400 and sundries \$1250, what was his net gain? What per cent of the cost was the net gain?

9. A merchant bought coal at \$6.25 per ton and sold it at \$7.50 per ton. What was his rate of gain?

10. A man bought a quarter section (160 acres) of government land at \$2.50 per acre. He sold $\frac{3}{4}$ of it at \$8.50 per acre and the remainder at \$10 per acre. The selling price was what per cent of advance over the first cost?

11. At the end of the year, a merchant found that his net profit was \$5000. His expenses were: rent \$720, clerk hire \$1500, advertising \$250, and sundry items \$650. If the average gross profit of his sales was $16\frac{2}{3}\%$ of the selling price, find the sales for the year.

12. A merchant bought sugar at $4\frac{1}{2}$ cents. If the loss in drying out, down weights, etc., was 10% of the amount bought, what per cent did he make by selling it at 6 cents?

13. A grocer bought 2000 bushels of potatoes at 45 cents. $\frac{2}{3}$ of them were sold at a gain of 20%, and the remainder at a loss of $11\frac{1}{3}\%$. Find the net rate of gain.

14. A merchant bought 400 barrels of apples at \$1.75 per barrel. He sold $\frac{3}{4}$ of them at \$2.50, $\frac{1}{8}$ of them at \$2.75, and the remainder at \$1.50. Find the net rate of gain.

15. After paying rent \$600, clerk hire \$2100, lighting and heating \$320, and sundries \$340, a man had a net profit of \$3900 from his business. If his sales for the year were \$64,000, find the average gross gain per cent upon the selling price of the goods. Find the net gain per cent upon the cost of the goods.

16. A real estate agent sold a farm of 145 acres at \$120 per acre, and the stock and utensils for \$3426. For his work he received 2% of the price of the farm and 3% of the price of the utensils. How much did he earn in all?

17. I paid a collector \$28.20 for collecting a note. If his charges were 5% of the amount collected, how much did he collect?

18. During the year, a merchant's sales amounted to \$24,650. The average gain was 20% of the sales. His rent was \$480, clerk hire \$1800, and sundries \$540. The net gain was what per cent of the cost of the goods?

19. If the wages of a certain class of workmen are to be raised 15% from their present scale, which is \$1.80 per day, what will the wages be under the new scale? How much will this amount to in a year to a man who works 285 days?

20. A dealer bought 25 boxes of oranges at \$3.50 per box of 144 each. In all, he found 120 decayed ones. At what price per dozen must he sell the remainder to gain 25% of the total cost?

21. A grocer bought 250 pounds of raw coffee bean at $14\frac{1}{2}$

cents a pound. In roasting, it lost 25% of its weight. He sold the roasted coffee at 22 cents a pound. How much did he make? What per cent of the cost did he make?

57. Commercial discount. It is a custom among certain wholesale dealers, manufacturers, and publishers to fix a price, called the **list price**, on their goods and then allow a certain deduction from this price to "the trade," *i. e.*, to retail dealers handling their kind of goods. This deduction is called **discount** and is usually a per cent of the list price.

ORAL PROBLEMS.

1. A bill of goods listed at \$90, discounted 20%, will cost what?

2. Goods listed at \$150 cost \$120. What was the rate of discount?

3. What can I discount the list price to make 20% profit on goods bought at 25% below the list price?

SUGGESTION. The question is, at what per cent of the list price must goods costing 75% of the list price be sold to gain 20% of the cost?

4. Goods bought at 10% below the list price and sold at 20% above the list price yield what per cent profit?

SUGGESTION. 80% of the list price is gained on what cost 90% of the list price. What part, then, of the cost is the gain?

5. Goods bought at 40% discount are sold for 20% below the list price. What is the per cent of profit?

6. A piano dealer bought a piano listed at \$600 for 50% off, and sold it for 30% off. How much did he make?

WRITTEN PROBLEMS.

1. A bill of goods listed at \$375 was sold at 22% off. Find the net price.

2. I got a discount of \$294 from a bill of goods listed at \$840. What was the rate of discount?

SOLUTION.

$$\begin{array}{r} .35 = 35\% \\ 840)294.00 \\ \underline{252.0} \\ 42.00 \\ \underline{42.00} \end{array}$$

EXPLANATION. Since it is required to find the rate per cent that one number is of the other, first find the ratio of \$294 to \$840, which is $\frac{294}{840}$, then express it as *hundredths*.

3. I bought goods that were listed at \$1150 for \$920. What rate of discount was this?

4. A bill of \$1980 was discounted $16\frac{2}{3}\%$. Find the net price. (Hint: $16\frac{2}{3}\% = \frac{1}{3}$.)

5. A merchant is usually allowed 30, 60, or sometimes 90 days, in which to pay for a bill of goods, but is allowed a discount for cash within 10 days. How much does a merchant save at "6% off for cash" on a bill of \$588?

6. During a single month a merchant received goods amounting to \$10,486. His average discount for cash was $4\frac{1}{2}\%$. How much did he save by paying cash?

7. A dealer sold goods listed at \$980 at a discount of $12\frac{1}{2}\%$ from the list price, and still made $16\frac{2}{3}\%$ on what they cost him. What discount did the dealer get from the list price?

8. I bought a bill of goods listed at \$1650 at a discount of 40%. What discount can I give from the list price to make 30% of what the goods cost me?

9. I made \$84 on a bill of goods. This was $16\frac{2}{3}\%$ of what they cost me. At what were they listed if I received a discount of 20% from the list price?

10. In making an estimate on a heating system, the plumber's terms were as follows: a boiler listed at \$240, discount 25%; radiators, \$84.50, discount 35%; fittings, \$37.50, discount 40%; work and other expenses \$150. Find the total cost.

11. A jobber sold a bill of goods for \$66.80, which was 15% less than the list price. The jobber got a discount of 25%

from the list price. How much did he make? What percent did he make?

12. A dealer bought 3 wagons listed at \$70 each, and 2 others listed at \$92 each. He received a discount of $47\frac{1}{2}\%$ from the list price. What was the net invoice price? By paying cash he got a further discount of 5% from the net invoice price. How much did he save by paying cash?

EXERCISES FOR DRILL.

	List Price	Rate of dis- count	Dis- count	Net Price		List Price	Rate of dis- count	Discount	Net Price
13.	\$ 465	20			20.	\$ 950		\$96	
14.	\$ 970	$33\frac{1}{4}$			21.	\$1050		\$175	
15.	\$1640	40			22.	\$1260			\$1050
16.	\$1830	5			23.	\$ 940		\$23.50	
17.	\$1960	$2\frac{1}{2}$			24.	\$ 630			\$598.50
18.	\$3470	5			25.	\$ 126			\$108
19.	\$1698	$16\frac{2}{3}$			26.			\$236.25	\$708.75

58. Successive discounts. When wholesale houses wish to increase the discount already given, they usually add another discount to be taken from the former discounted, or *net* price, rather than give a new *single* discount.

Thus, if goods are quoted at \$600, less 25% and 10%, it means that \$600 is to be discounted 25%, and the remainder, \$450, is then to be discounted 10%, leaving a net cost of \$405.

WRITTEN PROBLEMS.

1. A bill of hardware was listed at \$96, 40% and 10% off. Find the net price.

2. Find the net cost of a bill of \$85, 40% and 5% off.

3. Find the net cost of 8 doz. drip pans listed at \$4.45 per dozen, and 15 coal hods listed at \$2.10 per dozen; discounts 60%, 10%, and 10%.

4. Find the net cost of 24 doz. basting spoons at \$3.00 per dozen, and $\frac{1}{4}$ gross galvanized buckets at \$58 per gross; discounts 75% and 10%.

5. A dealer received a bill of window glass listed at \$730, but the discounts were 90% and 15%. Find the net cost.

6. A hardware dealer received an invoice of hardware listed at \$850. The discounts were 40% and 10%. Find the net cost.

7. A bill of chinaware listed at \$736 had discounts of 66 $\frac{2}{3}$ % and 10%. Find the net price including \$8.36 for boxing, freight, and drayage.

8. One-third of the gross amount of a bill of silverware amounting to \$846 was discounted at 40%, 10%, and 10%, and the remainder at 40% and 15%. Find the net amount of the bill. If the dealer retails the entire bill at an average of 90% of the list price, what does he make? What per cent of the net cost is this?

9. A dealer receives a bill the gross amount of which is \$334. The discounts are 40% and 10%. The freight, drayage, and sundry expenses amount to \$12.50. If the dealer receives an average of 85% of the list price, what per cent does he make on the net cost?

10. If a dealer gets discounts of 40% and 15%, what discount can he give on the list price to make 30% on his investment?

11. A jobber sold a plate glass for \$150, less discounts 50%, 30%, and 10%, and made a profit of 25% upon what it cost him. What did it cost the jobber?

12. A dealer received the following invoice of wagons: 3 listed at \$79 each; 2 listed at \$81 each; 4 listed at \$103 each; and one listed at \$85. The discounts were 40% and 5%. Find the net invoice. If a further discount of 5% for cash is given, what will he save by paying cash?

13. What per cent profit will the wagons described in Problem 12 yield, if bought for cash and sold at 10% from the list price?

14. A dealer bought disc harrows listed at \$18.50, at 35% and 10% off, and sold them through an agent at 10% off from the list price. The agent got 25% of the amount he received for the harrows. Find the net profit of both the dealer and the agent on each harrow.

59. **Billing goods.** Among the following problems are forms of bills rendered by wholesale dealers to retail dealers. A bill should show *the date that it was rendered*; the *date*, *price*, and *amount of each purchase*; the *terms*, i. e., the *discounts*, if any; and the *net amount* of the bill.

PROBLEMS.

1. What amount will settle the following bill, if paid within 10 days from the date of the bill?

WHITALL TATUM CO.

PHILADELPHIA, PA.

May 25, 1908.

Sold to J. R. WICK,

Covington, Ind.

Terms: { Net 60 days; 2½%
discount if paid
in 10 days.

2½	Gross tooth brushes No. 1064 at \$ 8.80	22	00		
½	" " " No. 8100 at \$12	2	00		
1½	" " " No. A-A at \$13.80	20	70		
		44	70		
	Less discount 40%, 10%			24	14
½	doz. spec. water bottles at \$18	9	00		
½	" " " " at \$20	10	00		
1	" " " " "	12	00		
		31	00		
	Less discount 20%, 5%			23	56
				47	70

2. Is the following bill correct? What will settle this bill before 5/13/'08?

Detroit, Mich., 5-8-'08.

G. H. GILMORE & Co., ROCHESTER, N. Y.

To PITTSBURGH PLATE GLASS COMPANY, Dr.

Terms: 2% 10 days, net 60 days.

Quantity	Size	Forwarded at Purchaser's Risk		Price			Total	
4 boxes	26×32	DAA	\$6.65	26	60			
6 "	24×28	"	5.81	81	86			
10 "	12×16	"	1.26	12	60			
		Less 90%, 20%		71	06	5 69		
2 "	26×30	DAA	9.03	18	06			
2 "	27½×40	"	10.81	20	62			
		Less 90%, 15%		38	68	8 29	8	98

3. Make out a bill from the same firm and to the same parties for the following: Date, March 5, 1908.

8—10¼×16½ SAA, at \$1.14; 5—21¼×26½ DAA, at \$5.27;

6—23¼×28 DAA, at \$6.02; 3—19¼×23 DAA, at \$4.80;

6—27¼×25¼ SAA, at \$6.76. Discount 90 and 15%.

4. Complete the following bill:

CHICAGO-KENOSHA HOSIER CO.

MANUFACTURERS OF SEAMLESS AND FULL-FASHIONED HOSIERY.

Sold to Messrs. J. R. Doe & Co., Decatur, Ill.

Terms { 2 per cent 10 days.
Net 80 days.

July 15, 1908.

No. of doz.	Description	Price			Totals
2½	Boys' Stockings	1.90			
1½	" "	2.00			
7½	" "	2.10			
4	" "	2.25			

If Doe & Co. remit before July 25, what should be the amount of the remittance? What must they remit any time between July 25 and Aug. 15?

5. Bill from the same firm to A. L. Dickson & Son, Jan. 4/08 the following: $3\frac{1}{2}$ doz. pr. hose at \$3.25; $7\frac{1}{2}$ doz. pr. children's stockings at \$1.80; $12\frac{1}{2}$ doz. pr. gent's half hose at \$2.50; $8\frac{1}{2}$ doz. pr. boys' stockings at \$2.25. What will settle the bill within 10 days? What within 30 days?

6. Complete the following:

Indianapolis, Ind., July 6, 1908.

MR. R. H. BRABB, Springfield, Mo.

Bought of HOLLWEG & REESE,

Importers of China, Glass & Queensware.

Terms: 60 days or 2% off 10 days.

6	Doz. 5274 Plates	\$2.25	*	*		
6	" " Teas	2.50	*	*		
$7\frac{1}{2}$	" " Coffee Cup Only	2.40	*	*		
6	" " Fruit	1.50	*	*		
2	" " Deep Coup. Soup	2.00	*	*		
1	Only " Covd. Dish	1.65	*	*		
3	" " Casseroles	1.65	*	*		
2	" " Boat Stant	.85	*	*		
3	" " 10 in. Dish	.50	*	*		
6	" " 12 " "	.83	*	*		
			*	*		
		Less 10%	*	*		
					*	*
2	Doz. 19783 H. & Co. Fruit	3.19	*	*		
2	Only " Baker	1.50	*	*		
2	" " Lobster Salad	1.69	*	*		
			*	*		
		Less 25%, 10%	*	*		
			*	*		
		Pkg.		50	*	*
					*	*

7. Check the following, that is, see if it is correct:

	Description	Price		Gross amount		Net amount	
$\frac{1}{4}$	doz. Alum. Trays & Scrapers	4	00			1	00
$\frac{1}{4}$	doz. " "	4	00			1	00
1	doz. Favorite Cake Spoons						75
1	doz. Galv. Tubs			5	70		
1	doz. " "			5	30		
$\frac{1}{2}$	doz. " "	7	20	3	60		
$\frac{1}{2}$	doz. " "	7	20	3	60		
1	doz. Skimmers				70		
1	doz. Rings				25		
$\frac{1}{8}$	gro. Dippers	7	80		65		
$\frac{1}{8}$	doz. Bread Raisers	4	80		80		
$\frac{1}{8}$	doz. " "	5	70		95		
$\frac{1}{8}$	doz. " "	6	90	1	15		
1	doz. Cup Dippers				65		
	Less 10%			24	35		
$\frac{1}{4}$	gro. Slop Jars	36	00	9	00	21	92
	Less 10%					8	10
1	doz. Dippers						55
2	doz. Basting Spoons, per gr.	22	00	3	67		
2	doz. " " " "	20	00	4	83		
2	doz. " " " "	36	00	6	00		
	Less 75-10%			14	50		
1	doz. Soup Strainers					3	27
1	doz. Pie Tins					1	00
$\frac{1}{8}$	gro. Dippers	11	40				45
							95
						38	99

Make out bills for the following :

8. Butler Bros., Chicago, Ill., sold to E. M. Rose & Co., Jan. 3, 1908, the following : 240 yd. matting at $17\frac{1}{2}$ cents ; 360 yd. matting at 18 cents ; 8 rugs at \$3 ; 2 matting rugs at \$0.78 ; 5 couch covers at \$3.25 ; 6 pr. curtains at \$1.50 ; 12 pr. curtains at \$2.75. Terms : 1 per cent in 20 days ; net 40 days.

What will settle the bill before Jan. 23? What from Jan. 23 to Feb. 12?

9. Sydney Shepherd & Co., Buffalo, N. Y., sold to H. M. Murphy & Son, March 13, 1908, the following:

$\frac{1}{4}$ gro. pails at \$39.60; $\frac{1}{2}$ gro. steamers at \$28.80; $\frac{1}{8}$ gro. steamers at \$32.40; all less 10%. Also $\frac{1}{4}$ gro. pails at \$31 net; $1\frac{1}{2}$ doz. drip pans at \$4.45 less 60, 10 and 10%; $\frac{1}{4}$ doz. bond boxes at \$5.10 less 10%. Terms: 2% off 10 days; net 60 days.

What will settle the bill before March 23? What, within 60 days?

10. Syracuse Chilled Plow Co., sold to Mr. Dawson, Feb. 8, '08, the following:

3 Jr. sulky plows at \$43 each; 7 No. 443 foot plows at \$16.50; 12 No. 402 foot plows at \$13.50; 4 disc harrows at \$27.75; and 8 sectional lever harrows at \$9.75. Discount 35% and 5%. Terms: 4 months, or 5% discount for cash in 30 days.

What will settle the bill in 30 days? In 4 months?

60. Marking at a profit and discounting marked price.

Sometimes goods marked to sell at a profit must be sold at a discount on the marked price. The problem then, is to find the net loss or gain.

ORAL PROBLEMS.

1. Goods costing \$2.50 were marked to sell at a gain of 40%, but were sold at 25% less than marked. Find the gain.

2. Shoes costing \$30 per dozen were marked to sell at a gain of 50%, but were sold for "a quarter off," that is, at a discount of 25% from the marked price. What was the gain or loss on each pair?

3. Boys' suits costing \$60 per dozen were marked to sell at a

gain of 40%, and sold at "a quarter off." What was gained or lost?

4. An article costing \$3.50 was marked at an advance of 20% and discounted 10%. Find the gain.

5. Goods marked to sell at 40% gain were discounted 10%. What per cent of the cost was gained?

SUGGESTION.—The marked price was 140% of the cost. The discount was 14% of the cost. Then the selling price was 126% of the cost.

6. Goods marked to sell at 20% gain were sold at 10% off. What per cent was gained?

7. What is lost by marking at an advance of 50% and discounting 40%?

8. What is gained by marking at an advance of 40% and discounting 20%?

9. A merchant marked his shoes to sell at a gain of 60%. What does he make on a pair marked at \$4.80 when selling at "a quarter off"?

10. A merchant made \$2.00 when selling an \$18 overcoat at "a quarter off." At what per cent of profit had he marked it?

11. What discount can a merchant give on the marked price of goods marked to gain 50% in order that he may get the cost of his goods?

12. After marking down a suit 25%, a dealer asked \$15 for it. Being unable to sell it at this price, he gave another discount of 15% (on the \$15) and still made 75 cents. At what per cent above cost was it first marked?

13. A dealer received an invoice of \$3500 worth of goods which he marked $33\frac{1}{3}\%$ above cost. He then marked them down 10% for a "bargain sale." What was his profit? What per cent of profit did he make?

DRILL EXERCISES.

No.	Cost Price	Rate of gain	Profit	Marked Price	Rate of Discount	Selling Price	Net loss or gain
1	\$ 3.50	20%			10%		
2	7.60	25%			12½%		
3	10.50	33⅓%			2½%		
4	8.60	33⅓%			10%		
5	9.60	16⅔%			5%		
6	16.40	20%			8%		
7	24.80	40%			25%		
8	16.50	45%			16⅔%		
9	17.20	50%			12½%		

No.	Marked rate of gain	Rate of Discount	Rate of loss or gain	No.	Marked rate of gain	Rate of Discount	Rate of loss or gain
10	50%	20%		19	30%	20%	
11	40%	10%		20	50%	33⅓%	
12	40%	16⅔%		21	40%	25%	
13	50%	16⅔%		22	30%	25%	
14	33⅓%	10%		23	20%	8%	
15	20%	12½%		24	25%	16⅔%	
16	40%	12½%		25	33⅓%	20%	
17	50%	8%		26	60%	40%	
18	60%	35%		27	60%	25%	

61. Selling on commission. Some agents, or salesmen, receive an amount equal to a certain per cent of their sales rather than a fixed salary. Salary thus obtained is called **commission**.

ORAL PROBLEMS.

1. What is the yearly income of a salesman who sells \$60,000 worth of goods at 7½% commission? If his expenses are \$150 per month, what is his net income?

2. A salesman earned \$3600 selling goods on a 6% commission. How much did he sell?

3. If \$6500 was the yearly commission from \$65,000 worth of goods, what was the rate?

4. A salesboy was offered his choice of the following: \$8.50 a week; \$5 a week and 1% of his sales; or 4% of his sales. He chose the last. His sales averaged \$265 per week. How much a week did he gain by his choice? How much is that for the year, if he worked 50 weeks?

DRILL EXERCISES (Oral).

	SALES.	RATE.	COM.		SALES.	RATE.	COM.
5.	\$85,000	4%		10.	\$345,000	2%	
6.	75,000	8½%		11.	95,000	12½%	
7.	96,000	6½%		12.	42,000	5%	
8.	24,000	25%		13.	65,000	15%	
9.	36,000	20%		14.	72,000	8%	

62. Buying through an agent. Sometimes an agent is employed to buy for another, receiving instead of a salary, a per cent of the cost of the goods.

ORAL PROBLEMS.

1. An agent bought 500 barrels of apples for me at \$1.26 per barrel. Find his commission at 3%. How much per barrel did it add to the cost?

2. My agent bought a car-load (720 bu.) of potatoes for me at 40 cents a bushel, commission 2½%. What was his commission? How much per bushel did it add to the cost? If freight, drayage and other expenses are 9 cents a bushel, for how much a bushel must I sell them to make 30%?

3. One month an agent bought 20,000 bu. of peaches at an average price of 50 cents. What did he earn at 2½% commission?

63. A commission merchant. A commission merchant is one who receives property from others for sale, usually receiving as compensation a per cent of the amount received.

NOTE. In case of car-load lots, the agent sometimes finds a buyer before the goods are shipped and they are sent direct to the buyer.

WRITTEN PROBLEMS.

1. A commission merchant sent me \$829 from the sale of 500 barrels of apples at \$1.75, after paying \$20 freight and \$8.50 storage. What commission did he charge me? How much per barrel was this?

2. I shipped produce to a commission merchant which he sold as follows: 68 crates of strawberries at \$2.56 per crate; 2,340 bunches of radishes at $2\frac{1}{4}$ cents a bunch; and 15 baskets of lettuce (18 pounds each) at 12 cents a pound. How much did I receive if I paid 5% commission?

3. A commission merchant sold a car-load of melons (1450) at 48 cents each, on a 5% commission. How much commission did he get from the sale?

NOTE.—A commission merchant who deals in hay, grain, and beans usually charges per car, ton, or bushel.

4. Complete the following account sales:

NEW YORK HAY EXCHANGE ASSOCIATION.

June 1, 1908.

Sold for account of M. DAWSON.

		Description.	No. Bales.	Weight.				
May	1	No. 1 timothy at \$20.	164	22,445	*	*		
"	7	" 8 " at \$16.	180	24,625	*	*		
"	7	" 1 clover at \$16.	170	23,440	*	*		
"	9	" 2 " at \$14.	160	21,435	*	*	**	*
Freight \$90.40. Drayage \$16.50. Storage \$45							**	*
Commission \$1 per ton.							**	*
Amount of draft							**	*

5. Mr. Mooreman paid the following prices for 3 car-loads of hay: No. 1 baled timothy \$14; No. 2 baled timothy \$12; No. 1 clover \$12. He shipped it to the Brooklyn Hay and Grain Co., who sold it as follows: 300 bales No. 1 timothy, 42,225 pounds at \$20; 120 bales No. 2 timothy, 16,345 pounds at \$18; 90 bales No. 1 clover, 13,440 pounds at \$17. Commission \$1 per ton. Freight and drayage \$96.75. Storage \$36. Find the net gain and the gain per cent.

6. A dealer paid \$1.80 per bushel (60 lb.) for beans, put them in bags of 165 pounds each, and sold them per car-load f. o. b. (free on board cars) at \$2.25 per bushel, through a commission merchant. The bags cost \$165 per 1000. The commission merchant's fee was 5 cents per bag. Find the net profit on a car-load of 320 bags.

7. A dealer bought 3 car-loads of wheat (3460 bu.) paying an average price of 82 cents. It was sold by a commission merchant for 86 cents (f. o. b.). Find the net gain, commission being \$5 per car.

NOTE.—Dairy and poultry products are usually sold on a 5% commission.

8. A commission merchant sells 1580 lb. of chickens at $8\frac{1}{4}$ cents a pound, and 56 cases of eggs (30 doz. each) at $18\frac{3}{4}$ cents a dozen. He charges 5% and deducts \$7.25 for freight and other charges. How much does he remit?

9. Find the net proceeds and the net gain from the following sale: 120 cases of eggs (30 doz. each) at 26 cents, 730 eggs broken; 60 crates of spring chickens, averaging 90 lb. per crate at 22 cents; and 40 tubs dairy butter, 80 lb. each, at 18 cents. Freight \$86. Drayage \$32. Storage \$28. Commission 5%. The average price paid was eggs, 18 cents; chickens, 16 cents; butter, 16 cents.

64. A broker. A broker negotiates sales or makes purchases for another, as an agent, for a commission on the amount sold or purchased.

The nature of his business is usually denoted by a prefix, as, stock-broker, cotton-broker, exchange-broker, etc. The difference between a commission merchant and a broker is, that the former actually handles that which is bought and sold, while the latter does not.

PROBLEMS.

1. A speculator buys 50,000 bu. of wheat, through a broker, for $86\frac{1}{8}$, and sells at $93\frac{1}{4}$. What does he make, brokerage being $\frac{1}{8}$ ct. per bushel for buying and for selling? What is the broker's fee?

2. If a speculator buys 1500 barrels of pork at \$14.05 and sells it at \$12.87 $\frac{1}{2}$, what is his total loss, not including interest, brokerage being $2\frac{1}{2}$ ct. per barrel for buying and the same for selling? What is the broker's fee?

3. What will 3500 bales (500 lb. each) of cotton cost at 9.6ct. per pound, brokerage 5ct. per bale? What is the brokerage?

4. A speculator buys 2800 bales of cotton at 10.2ct. and sells at 9.8ct. What is the total loss, brokerage 5ct. per bale for each transaction? What is the broker's fee?

CHAPTER III.

BORROWING, LOANING, AND INVESTING MONEY.

65. Interest. Money paid for the use of money is *interest*. It is a certain per cent of the amount borrowed, called the **principal**. The rate is usually for the use for a year, even though it may be collected semi-annually or quarterly.

66. A promissory note. One who borrows money from another gives a written promise, called a *promissory note*, to repay the money at a given time. The rate of interest also is stated in the note, unless the interest is paid in advance.

The following is a standard form of promissory note :

Indianapolis, Ind., <i>Apr. 3, 1908.</i>	
<i>\$500.</i>	
<i>Four months</i> after date I promise to pay to the order of	
<i>Henry S. Walker.....</i>	
<i>Five hundred and $\frac{07}{100}$.....</i>	Dollars,
value received, with interest at 6%.	
<i>John B. Anderson.</i>	

In case the note is given to a bank and the interest is paid in advance, the note is made payable to the cashier of the bank and no rate is named.

The following is a common form of such a note :

Ypsilanti, Mich., <i>Feb. 10, 1908.</i>	
<i>Ninety days</i> after date, I promise to pay to the order	
of R. W. Hemphill, Cashier,	<i>\$600</i>
<i>Six hundred</i>	Dollars
at Ypsilanti Savings Bank, Ypsilanti, Mich. Value received.	
Residence <i>314 Elm St.</i>	
Due <i>April 10.</i> Discount <i>\$9.00.</i>	<i>J. L. Turner.</i>

67. Security. One who loans money wishes, of course, to become reasonably secure from loss, that is, reasonably sure that the money will be repaid. If the financial standing of the borrower is good, it may be that his own signature is sufficient security. Sometimes two or more sign the same note. Any one of those who sign the note thus becomes responsible for its payment. This method is known as **personal security**. Sometimes the repayment is secured by the borrower making over certain real estate or personal property to the one of whom the money is borrowed. This is called a **mortgage**, and becomes void when the money is repaid.

68. Simple interest and bank discount. When money is loaned for long periods, the interest usually is collected at the *end* of each year (or at the end of each half-year, or each quarter, if so specified in the note). This is **simple interest**. Banks usually require the interest to be paid in advance, especially for money loaned for short periods, as 30, 60, 90, or 120 days. This interest paid to banks is called **Bank discount**.

Thus, if one should borrow \$100 at 6% for 3 months, in the case of *simple interest*, he would give his note for \$100 with interest at 6%. He has the use

of \$100 for 3 months and, at the end of that time, pays \$101.50.* When borrowing the same sum at a bank he gives his note for \$100 *without interest*, and pays the interest of \$1.50 from the \$100 in advance, leaving but \$98.50, called the *proceeds* of the note. At the end of the term, the \$100 is paid to the bank. Thus it is seen that the only difference between the *bank discount* and the *simple interest* is that in one case the interest of \$1.50 is paid when the money is borrowed, and in the other, when it is repaid. That is,

I. *Simple interest is a per cent of the money actually borrowed and used;*

II. *Bank discount is a per cent of the maturity value of the note.*

69. Date of maturity. To find the date upon which a note is due, the usual custom is to count forward the number of months or days according as months or days are stated in the note. 30 days are considered an interest month, and 360 days, a year.

70. Interest problems. The only problem with which the business man is concerned is to find the interest (or *amount*, i. e. interest plus the principal) when the time and rate are given.

To obtain the interest for any period of time, multiply the interest for one year by the time in years.

DRILL EXERCISES IN INTEREST AND BANK DISCOUNT.

Oral.

1. At 5%, what is the interest of \$400 for 1 year? For 6 months? For 2 years? For 2 yr. 6 mo.?

2. At 6%, what is a year's interest of \$500? 3 years'?

3. At 5%, what is 3 years' interest of \$800? 5 years'?

* It is becoming customary, especially among bankers, even where the interest is charged upon the amount actually received by the borrower, to add the interest to the face of the note rather than to name the rate. Thus, the interest of a \$700 note to run one year at 6% is \$42. Hence, a non-interest bearing note of \$742 is given.

At 6%, what is the interest of:

- | | |
|---------------------|------------------------------------|
| 4. \$500 for 6 mo.? | 6. \$250 for 1 year? |
| 5. \$300 for 8 mo.? | 7. \$150 for $2\frac{1}{2}$ years? |

At 5%, what is the interest of:

- | | |
|----------------------------|----------------------------|
| 8. \$800 for 1 yr. 6 mo.? | 11. \$400 for 2 yr. 3 mo.? |
| 9. \$200 for 2 yr. 6 mo.? | 12. \$800 for 1 yr. 9 mo.? |
| 10. \$300 for 2 yr. 8 mo.? | 13. \$700 for 4 years? |

Written.

1. At 5%, what is the interest of \$540 for 2 yr. 7 mo.?

SOLUTION.

$$\frac{31}{12} \times \frac{5}{100} \times \$540, \text{ or}$$

$$\frac{31 \times 5 \times \$540}{12 \times 100} = \$69.75$$

EXPLANATION. .05 of \$540 = the int. for

1 yr. Now 2 yr. 7 mo. = $2\frac{7}{12}$ yr. Hence the total interest = $2\frac{7}{12} \times .05 \times \540 .

NOTE. This is called the **general method** of finding interest.

Find the interest and amount of:

2. \$5000 at 5% for 1 yr. 3 mo.
3. \$6040 at 4% for 1 yr. 8 mo.
4. \$7500 at $4\frac{1}{2}$ % for 2 yr. 2 mo.
5. \$9460 at 6% for 6 mo. 12 da.
6. \$1350 at 5% for 8 mo. 20 da.
7. \$3460 at $4\frac{1}{2}$ % for 9 mo. 10 da.

Find the bank discount and proceeds of:

- | | |
|---|------------------------------|
| 8. \$4860 for 90 da. at 4%. | 11. \$3050 for 66 da. at 6%. |
| 9. \$6500 for 70 da. at 5%. | 12. \$5100 for 65 da. at 5%. |
| 10. \$1400 for 110 da. at $4\frac{1}{2}$ %. | 13. \$1950 for 72 da. at 4%. |

71. A short method of finding interest. Money is often borrowed for short periods, say 30, 60, or 90 days, especially at

banks. 6% is a very usual rate. In such cases, much time may be saved by observing that

Since the int. of any prin. for 360 da. at 6% is .06 of the prin., then the int. of any prin. for 60 da. at 6% is .01 of the prin.

EXAMPLE. Find the interest of \$720 at 6% for 90 days.

\$7.20
3.60

\$10.80

EXPLANATION. Since $\frac{1}{100}$ of \$720, or \$7.20, is the interest for 60 days, \$3.60 must be the interest for 30 days, and their sum the required interest.

DRILL EXERCISES.

At sight, give the interest at 6% of:

- | | | |
|----------------------|-----------------------|-----------------------|
| 1. \$540 for 60 da. | 7. \$120 for 45 da. | 13. \$2400 for 45 da |
| 2. \$600 for 90 da. | 8. \$750 for 40 da. | 14. \$3600 for 10 da |
| 3. \$840 for 30 da. | 9. \$630 for 120 da. | 15. \$4200 for 70 da. |
| 4. \$720 for 10 da. | 10. \$305 for 90 da. | 16. \$5400 for 50 da |
| 5. \$900 for 120 da. | 11. \$1200 for 15 da. | 17. \$6300 for 50 da |
| 6. \$540 for 90 da. | 12. \$1600 for 75 da. | 18. \$2100 for 20 da |

Find the interest at 6% of:

- | | |
|-------------------------|------------------------|
| 19. \$720 for 87 days. | 22. \$1360 for 33 da. |
| | 23. \$1540 for 54 da. |
| \$ 7.20=int. for 60 da. | 24. \$1680 for 93 da. |
| 2.40= " " 20 " | 25. \$2750 for 63 da. |
| .60= " " 5 " | 26. \$742 for 72 da. |
| .24= " " 2 " | 27. \$960 for 84 da. |
| <hr/> \$10.44= " " 87 " | 28. \$1432 for 36 da. |
| 20. \$965 for 117 da. | 29. \$1520 for 108 da. |
| 21. \$1050 for 96 da. | |

30. Find the interest of \$840 for 75 da. at 5%.

At 5%, find the interest of:

$$\begin{array}{r}
 \$8.40 = \text{int. at } 6\% \text{ for } 60 \text{ da.} \\
 2.10 = \text{ " " " " } 15 \text{ " } \\
 \hline
 6)10.50 = \text{ " " " " } 75 \text{ " } \\
 1.75 = \text{ " " } 1\% \text{ " " " } \\
 \hline
 \$8.75 = \text{ " " } 5\% \text{ " " " }
 \end{array}$$

31. \$820 for 70 da.

32. \$950 for 63 da.

33. \$720 for 93 da.

34. \$875 for 72 da.

35. \$970 for 85 da.

At 6%, find the amount of:

36. \$450 for 20 da. 39. \$650 for 45 da. 42. \$980 for 80 da.

37. \$120 for 50 da. 40. \$810 for 75 da. 43. \$735 for 76 da.

38. \$830 for 45 da. 41. \$960 for 70 da. 44. \$963 for 86 da.

72. Negotiable paper. Any kind of security, as a note, a bond, or any other written obligation, that may be transferred from one person to another is called a **negotiable paper**. A promissory note is negotiable when properly **indorsed**, that is, when signed across the back by the holder. Unless the indorser writes the words "without recourse" over his name, he is held responsible for the payment of the note in case it cannot be collected from the maker.

73. Selling notes. Business firms very frequently take notes due in 30, 60, or 90 days, or for any term agreed upon, either with or without interest, from their customers. A firm may merely indorse these notes in case the firm is of high financial standing (if not they may be asked for further security) and sell them to a bank, that is, deposit them at a bank and receive credit for the *proceeds*. It was seen in § 68 that a bank charges interest upon the maturity value of the note. Hence,

In buying a note (discounting), a bank computes the amount due at maturity, and charges interest on this amount from the day the note is bought until it is due.

NOTE. Banks compute interest on the nearest dollar, \$0.50 being

counted an extra dollar. Thus, to compute the interest on \$425.65, use \$426. They also usually compute the interest for the exact number of days the note has to run.

DRILL EXERCISES IN SELLING NOTES.

1. A note of \$600, dated May 3, 1908, interest 6%, to run 4 months, was discounted May 20, at 6%. Find the proceeds.

SOLUTION.—Amount of the note when due, Sept. 3 = \$612.

Interest on \$612 at 6% for 106 days (from May 20 to Sept. 8) = \$10.81.

Proceeds, or \$612 — \$10.81 = \$601.19.

2. A note of \$560 to run 3 months, dated Aug. 4, 1908, bearing 5% interest, was discounted at 6% on Sept. 19. Find the discount and the proceeds.

3. A man sold his farm for \$6500, taking a note due in 6 months, interest 5%. He at once sold the note to a bank, discount 6%. What did he get in cash for his farm?

4. A certain wholesale house sells goods on 90 days' time, taking in payment a 90-day note without interest. What does the house get for an invoice of \$820 discounted at 6%, 5 days after the date of the note?

5. In order to get cash for goods, some firms allow a discount of 2% from the bill, if paid within 10 days. Others take a 60-day or 90-day note which they discount if they need cash. On a bill of \$750, which is better for a firm, and how much, to allow a 2% discount or to discount a 60-day note at 6%? Which is better, 2% discount or to discount a 90-day note at 6%?

6. Some firms date their bills for certain kinds of goods two or three months ahead of the actual date of making out the invoice. A bill of wall-paper amounting to \$540 was billed Dec. 15, 1907, but dated March 1, 1908. If this bill is paid before March 1, a discount equal to 6% interest may be deducted. What will settle the bill Dec. 15? What will settle it Jan. 1, 1908? What, Feb. 1?

7. A note of \$730 to run one year, interest 6%, was discounted 54 days before it was due, at 6%. What were the discount and the proceeds?

74. Finding the difference between dates. Bank discount is usually reckoned for the exact number of days a note has to run. This time is found by the use of a table.

THIS TABLE SHOWS THE NUMBER OF DAYS FROM ANY DAY OF ANY MONTH TO THE SAME DAY OF ANY MONTH NOT MORE THAN ONE YEAR LATER.

From	To Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	30	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

EXAMPLE. Find the number of days from June 13 to Nov. 27 of the same year.

By the table, from June 13 to Nov. 13=153 days

From Nov. 13 to Nov. 27= 14 "

Total 167 "

75. Calculating interest by tables. By the use of tables, the computation of interest is reduced to addition. The next page is a specimen page from a book of interest tables.

INTEREST TABLE. INTEREST AT 6%

75

Da.	\$ 100	\$ 200	\$ 300	\$ 400	\$ 500	\$ 600	\$ 700	\$ 800	\$ 900	\$ 1000	Da.
1	0.017	0.033	0.050	0.067	0.083	0.100	0.117	0.133	0.150	0.167	1
2	0.033	0.067	0.100	0.133	0.167	0.200	0.233	0.267	0.300	0.333	2
3	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	3
4	0.067	0.133	0.200	0.267	0.333	0.400	0.467	0.533	0.600	0.667	4
5	0.083	0.167	0.250	0.333	0.417	0.500	0.583	0.667	0.750	0.833	5
6	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000	6
7	0.117	0.233	0.350	0.467	0.583	0.700	0.817	0.933	1.050	1.167	7
8	0.133	0.267	0.400	0.533	0.667	0.800	0.933	1.067	1.200	1.333	8
9	0.150	0.300	0.450	0.600	0.750	0.900	1.050	1.200	1.350	1.500	9
10	0.167	0.333	0.500	0.667	0.833	1.000	1.167	1.333	1.500	1.667	10
11	0.183	0.367	0.550	0.733	0.917	1.100	1.283	1.467	1.650	1.833	11
12	0.200	0.400	0.600	0.800	1.000	1.200	1.400	1.600	1.800	2.000	12
13	0.217	0.433	0.650	0.867	1.083	1.300	1.517	1.733	1.950	2.167	13
14	0.233	0.467	0.700	0.933	1.167	1.400	1.633	1.867	2.100	2.333	14
15	0.250	0.500	0.750	1.000	1.250	1.500	1.750	2.000	2.250	2.500	15
16	0.267	0.533	0.800	1.067	1.333	1.600	1.867	2.133	2.400	2.667	16
17	0.283	0.567	0.850	1.133	1.417	1.700	1.983	2.267	2.550	2.833	17
18	0.300	0.600	0.900	1.200	1.500	1.800	2.100	2.400	2.700	3.000	18
19	0.317	0.633	0.950	1.267	1.583	1.900	2.217	2.533	2.850	3.167	19
20	0.333	0.667	1.000	1.333	1.667	2.000	2.333	2.667	3.000	3.333	20
21	0.350	0.700	1.050	1.400	1.750	2.100	2.450	2.800	3.150	3.500	21
22	0.367	0.733	1.100	1.467	1.833	2.200	2.567	2.933	3.300	3.667	22
23	0.383	0.767	1.150	1.533	1.917	2.300	2.683	3.067	3.450	3.833	23
24	0.400	0.800	1.200	1.600	2.000	2.400	2.800	3.200	3.600	4.000	24
25	0.417	0.833	1.250	1.667	2.083	2.500	2.917	3.333	3.750	4.167	25
26	0.433	0.867	1.300	1.733	2.167	2.600	3.033	3.467	3.900	4.333	26
27	0.450	0.900	1.350	1.800	2.250	2.700	3.150	3.600	4.050	4.500	27
28	0.467	0.933	1.400	1.867	2.333	2.800	3.267	3.733	4.200	4.667	28
29	0.483	0.967	1.450	1.933	2.417	2.900	3.383	3.867	4.350	4.833	29
Mo.	\$ 100	\$ 200	\$ 300	\$ 400	\$ 500	\$ 600	\$ 700	\$ 800	\$ 900	\$ 1000	Mo.
1	0.500	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	5.000	1
2	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000	2
3	1.500	3.000	4.500	6.000	7.500	9.000	10.500	12.000	13.500	15.000	3
4	2.000	4.000	6.000	8.000	10.000	12.000	14.000	16.000	18.000	20.000	4
5	2.500	5.000	7.500	10.000	12.500	15.000	17.500	20.000	22.500	25.000	5
6	3.000	6.000	9.000	12.000	15.000	18.000	21.000	24.000	27.000	30.000	6
7	3.500	7.000	10.500	14.000	17.500	21.000	24.500	28.000	31.500	35.000	7
8	4.000	8.000	12.000	16.000	20.000	24.000	28.000	32.000	36.000	40.000	8
9	4.500	9.000	13.500	18.000	22.500	27.000	31.500	36.000	40.500	45.000	9
10	5.000	10.000	15.000	20.000	25.000	30.000	35.000	40.000	45.000	50.000	10
11	5.500	11.000	16.500	22.000	27.500	33.000	38.500	44.000	49.500	55.000	11
Yr.	\$ 100	\$ 200	\$ 300	\$ 400	\$ 500	\$ 600	\$ 700	\$ 800	\$ 900	\$ 1000	Yr.
1	6.00	12.00	18.00	24.00	30.00	36.00	42.00	48.00	54.00	60.00	1
2	12.00	24.00	36.00	48.00	60.00	72.00	84.00	96.00	108.00	120.00	2
3	18.00	36.00	54.00	72.00	90.00	108.00	126.00	144.00	162.00	180.00	3
4	24.00	48.00	72.00	96.00	120.00	144.00	168.00	192.00	216.00	240.00	4
5	30.00	60.00	90.00	120.00	150.00	180.00	210.00	240.00	270.00	300.00	5

EXAMPLE 1. Find the bank discount on \$240 at 6% from March 12, 1908, to July 25, 1908.

By the tables, from March 12 to July 12 = 122 days

From July 12 to July 25 = 13 days

Total = 135 days, or 4 mon. 15 days.

By the tables, Int. of \$200 for 15 days = \$0.500

" " 40 " " " = 0.100

" " 200 " 4 mon. = 4.000

" " 40 " " " = 0.800

5.40

EXAMPLE 2. Find the bank discount on \$670 at 6% from May 18 to Sept. 2.

By the tables, from May 18 to Sept. 18 = 123 days

" Sept. 2 to " " = 16 "

Total 107 days, or 3 mo. 17 days

Int. of \$600 for 17 days = \$1.700

" " 70 " " " = 0.198

" " 600 " 3 mo. = 9.000

" " 70 " " " = 1.050

\$11.95

DRILL EXERCISES IN USING THE TABLES.

By the tables, find the interest at 6% of:

- | | | |
|----------------------|-----------------------|-----------------------|
| 1. \$850 for 18 da. | 7. \$5600 for 83 da. | 13. \$972 for 41 da. |
| 2. \$970 for 33 da. | 8. \$7200 for 91 da. | 14. \$895 for 111 da. |
| 3. \$680 for 42 da. | 9. \$8600 for 113 da. | 15. \$908 for 117 da. |
| 4. \$910 for 47 da. | 10. \$7840 for 69 da. | 16. \$675 for 175 da. |
| 5. \$780 for 68 da. | 11. \$645 for 27 da. | 17. \$861 for 134 da. |
| 6. \$3400 for 71 da. | 12. \$837 for 39 da. | 18. \$276 for 142 da. |

Find the bank discount and proceeds of:

19. \$450 from Aug. 6, 1907, to Nov. 15, 1907, at 6%.

20. \$960 from Jan. 3, 1908, to June 18, 1908, at 6%.

21. \$4300 from March 18, 1908, to June 3, 1908, at 6%.

22. \$1850 from Oct. 1, 1907, to Jan. 19, 1908, at 6%.

23. \$3800 from Aug. 30, 1908, to Nov. 3, 1908, at 6%.

24. \$4250 from Nov. 24, 1907, to Feb. 6, 1908, at 6%.

76. Partial payments on a note. In general, modern business custom will not allow an advance payment upon a note unless it is so stipulated in the note. Such stipulations are usually to the effect that such payments are to be made at the end of interest paying periods. There was a time, however, when borrowing among individuals was more common than now, and when the payments of interest and any other payments were more irregular. To govern the final settlements in such transactions, the United States Supreme Court decreed that

Partial payments of notes must first be used to cancel the interest due. Any balance remaining may be used to lessen the principal. If, however, the payment is too small to pay the interest due, the unpaid interest must not be used to increase the principal, which must never represent more than the money actually and previously due.

PROBLEMS.

1. What is due Jan. 16, 1908, on a note of \$1600 drawing 6% interest, given May 3, 1904, the following payments having been made: May 3, 1905, \$200; Dec. 18, 1905, \$80; June 25, 1906, \$300; April 16, 1907, \$450?

2. What is due July 23, 1908, on a note of \$2400 drawing 5% interest, given Jan. 4, 1903, the following payments having been made: Sept. 4, 1903, \$300; July 19, 1904, \$50; Jan. 4, 1905, \$300; April 16, 1906, \$250; Dec. 26, 1907, \$500?

NOTE.—Since the payment on July 19, 1904, was insufficient to pay the interest then due, the principal left on Sept 4, 1903, will have to be used again. Work will be saved by finding the amount from Sept. 4, 1903, to Jan. 4, 1905, and then deducting \$350, or both payments.

3. A note of \$3500, dated Aug. 15, 1905, interest 6%, has the following endorsements: Aug. 15, 1906, \$500; Feb. 15, 1907, \$100; July 10, 1907, \$400; Dec. 15, 1907, \$1500. What is due Aug. 15, 1908?

4. A note of \$7500, dated May 3, 1905, interest 5%, has the following endorsements: May 3, 1906, \$2500; Aug. 3, 1906, \$3500; Feb. 18, 1907, \$50; May 3, 1907, \$500. What is due May 3, 1908?

77. Savings banks. A savings bank is an institution for receiving and investing savings. It pays interest at stated intervals, usually semi-annually, upon the deposits. As the bank's profits depend upon the difference between the rate of interest paid and the rate received from loans, it pays a low rate, usually, 3, $3\frac{1}{2}$ or 4%.

78. Compound interest. When interest due at the end of any interest period is added to the principal and thus draws interest for the next interest period, and so on, we have **compound interest**. Thus, in a savings bank, the interest due at any interest paying date is credited to one's account and thus draws interest, giving *compound interest*.

NOTE.—It is interesting and surprising to know that one cent at 10% simple interest from the beginning of the Christian Era to the present time (1908) would amount to but \$1.92 while compounded annually it would amount to more than $\$6 \times 10^{15}$.

79. Use of compound interest. In most states the collection of compound interest is illegal. In modern practice, then, the subject is merely of use to large investors, as building and loan associations, life insurance companies, banking corporations, etc., who wish to compute the final incomes from reinvesting all interest as it falls due. Such computations are made by the use of compound interest tables.

A SECTION OF A COMPOUND INTEREST TABLE

79

PERIODS.	1 per cent.	1½ per cent.	2 per cent.	2½ per cent.	3 p. cent.	PERIODS.
1	1.010000	1.015000	1.020000	1.025000	1.030000	1
2	1.020100	1.030225	1.040400	1.050625	1.060900	2
3	1.030301	1.045678	1.061208	1.076891	1.092727	3
4	1.040604	1.061364	1.082432	1.103813	1.125509	4
5	1.051010	1.077284	1.104081	1.121408	1.159274	5
6	1.061520	1.093443	1.126162	1.159693	1.194052	6
7	1.072135	1.109845	1.148686	1.188686	1.229874	7
8	1.082857	1.126493	1.171660	1.218403	1.266770	8
9	1.093685	1.143390	1.195093	1.248863	1.304773	9
10	1.104622	1.160541	1.218994	1.280085	1.343916	10
11	1.115668	1.177949	1.243374	1.312087	1.384234	11
12	1.126825	1.195618	1.268242	1.344889	1.425761	12
13	1.138093	1.213552	1.293607	1.378511	1.468534	13
14	1.149474	1.231756	1.319479	1.412974	1.512590	14
15	1.160969	1.250232	1.345868	1.448293	1.557967	15
16	1.172579	1.268985	1.372786	1.484506	1.604706	16
17	1.184304	1.288020	1.400241	1.521618	1.652847	17
18	1.196147	1.307341	1.428246	1.559659	1.702433	18
19	1.208109	1.326951	1.456811	1.598650	1.753506	19
20	1.220190	1.346855	1.485947	1.638616	1.806111	20

PERIODS.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	6 p. cent.	PERIODS.
1	1.035000	1.040000	1.045000	1.050000	1.060000	1
2	1.071225	1.081600	1.092025	1.102500	1.123600	2
3	1.108718	1.124864	1.141166	1.157625	1.191016	3
4	1.147523	1.169859	1.192518	1.215506	1.262477	4
5	1.187686	1.216653	1.246181	1.276281	1.338226	5
6	1.229255	1.265319	1.302260	1.340096	1.418519	6
7	1.272279	1.315932	1.360861	1.407100	1.503630	7
8	1.316809	1.368569	1.422100	1.477455	1.593848	8
9	1.362900	1.423312	1.486095	1.551323	1.689479	9
10	1.410600	1.480244	1.552969	1.628895	1.790848	10
11	1.459970	1.539454	1.622853	1.710339	1.898299	11
12	1.511069	1.601032	1.695881	1.795856	2.012197	12
13	1.563956	1.665074	1.772196	1.885649	2.132923	13
14	1.618695	1.731676	1.851945	1.979931	2.260904	14
15	1.675348	1.800944	1.935282	2.078923	2.396558	15
16	1.733986	1.872981	2.022370	2.182875	2.540351	16
17	1.794676	1.947901	2.113376	2.292018	2.692773	17
18	1.857489	2.025817	2.208478	2.406619	2.854339	18
19	1.922501	2.106849	2.307860	2.526950	3.025600	19
20	1.989789	2.191123	2.411714	2.653298	3.207136	20

PROBLEMS.

1. If a man deposits \$500 semi-annually in a savings bank, what amount will he have to his credit at the end of 5 years, payable semi-annually, interest 3%?

SOLUTION.

$$\begin{array}{r}
 \$1.015000 \times 500 \\
 1.080225 \times 500 \\
 1.045678 \times 500 \\
 1.061364 \times 500 \\
 1.077284 \times 500 \\
 1.098448 \times 500 \\
 1.109845 \times 500 \\
 1.126493 \times 500 \\
 1.148390 \times 500 \\
 1.160541 \times 500
 \end{array}$$

EXPLANATION—This rate is $1\frac{1}{2}\%$ for 10 periods. The last deposit draws interest for but one period of 6 months; the next for 2 periods, and so on to 10 periods. Work is saved by adding before multiplying by 500.

$$\$10.863263 \times 500 = \$5431.63$$

2. How much can one accumulate in 10 years by depositing \$800 annually, interest 4%, compounded annually?

3. To what will an annual payment of \$564 for 10 years amount, interest $3\frac{1}{2}\%$, compounded annually?

4. What may one draw at the end of 10 years if he regularly deposits \$400 semi-annually in a savings bank paying 4%, payable semi-annually?

5. If one can invest \$600 yearly from his salary where it will yield him 6% yearly, and can keep all the interest invested at the same rate as fast as it accrues, how much can he accumulate in 20 years?

6. On his sixth birthday and on each succeeding birthday including his 21st, a man gave his son \$100, which together with the interest, was kept in a savings bank paying 4% interest, annually. How much had the young man to his credit from this source on his 21st birthday?

80. Stock investments. Another way to invest one's money is in stock in some corporation.

81. A corporation consists of a number of individuals united

in one body or company, and empowered to act in a certain capacity, or to transact business of some form or nature. A **stock corporation** is a corporation the ownership of whose stock is divided into **shares**. A **municipal corporation** is a corporation organized for the purpose of local government, as a city or town.

82. A stock certificate is a formal statement issued by a corporation stating the number of shares of stock of which the holder is owner.

A STOCK CERTIFICATE.

No. <i>342</i>	50 Shares
<i>Incorporated under the Laws of the State of Michigan.</i> ACME PORTLAND CEMENT COMPANY JACKSON, Mich, <i>3, 8, 08.</i> This certifies that <i>B. N. Mason</i> is the owner of <i>Fifty</i> shares, of One Hundred Dollars each, of the capital stock of the Acme Portland Cement Company. Transferable only on the books of the company in person or by attorney upon surrender of this certificate. <i>C. R. Snyder.</i> Secretary. <i>J. M. Dixon.</i> President.	

83. Par value. The value of each share of stock as named in the certificate is the *par value*. The par value is determined by the corporation, and depends upon the number of shares into which it seems desirable to divide the stock.

Thus, if the capital stock of a corporation is \$100,000 and there are 1000 shares, the *par value* of each share is \$100. If there are 2000 shares in the corporation, the par value of each is \$50.

84. The market value of stock does not depend upon the

par value, but upon the earnings of the company. The *market quotations* are for a par value of \$100.

Thus, C. & N. W. quoted at 184 $\frac{1}{2}$ means that stock whose par value is \$100 is selling for \$184.75.

85. Dividends. The earnings of a corporation that are divided among its stockholders are called the **dividends**.

86. Kinds of stock. Two kinds of stock are issued, **common stock** and **preferred stock**. The rate of dividends of the preferred stock (usually from 5% to 7%) is fixed by the corporation, before the stock is sold. The dividends of the preferred stock are paid from the earnings before those on the common stock are paid. After the dividends on the preferred stock are paid, a part or all of the remainder of the net earnings is divided among the holders of the common stock as a certain per cent of the par value of the stock held.

87. Buying stock. There are two classes of buyers of stock, viz., the **investor** who buys stock and holds it for the dividends that it yields, and the **speculator** who buys expecting the price to advance so that he may sell at a profit.

When buying as an *investment*, one must bear in mind that there is an element of uncertainty as to the dividends that will be declared upon stock; and also that stock is subject to great fluctuations in the market. It is not, then, a desirable form of investment for one whose support depends entirely upon the income, or for the trustee of an estate that is to provide a regular stated income. (See table on next page.)

88. A stock broker. In general, one wanting to buy stock in a particular corporation does not know who has it for sale, or if he wishes to sell, does not know of a buyer; hence, he has to buy or sell through an agent called a *stock broker*, who usually belongs to an association called a **stock exchange**. The broker usually gets $\frac{1}{2}$ % of the par value for buying, and the same for selling.

THE FOLLOWING TABLE WILL SHOW SOME FLUCTUATIONS OF THE MARKET:
RAILROAD STOCKS.

NAME OF THE STOCK	Highest price in 1906	Highest price in 1907	Closing price Nov. 23, 1907	Closing price Dec. 24, 1907	Closing price Jan. 27, 1908	Closing price Feb. 22, 1908	Closing price March 13, 1908	Last Annual Dividend
Atchison. . . .	110½	108½	70½	70½	72½	68½	71½	6%
Baltimore & Ohio.	125½	122	78½	82	87½	79½	80½	6%
Chi. M. & St. P.	199½	157½	97½	103	112½	108½	116	7%
Great Northern. .	348	189½	113½	115	122	118	121	7%
Reading. . . .	164	139½	81½	92½	101½	95½	102½	4%
Union Pacific. .	195½	183	112½	116½	122½	116½	122	10%

INDUSTRIAL STOCKS.

Am. Sugar (com.)	157	137½	103½	98½	113½	112½	118	7%
U. S. Steel (com.)	50½	50½	24	25½	28½	28½	32½	2%
U. S. Steel (pfd.)	113½	107½	81½	87½	92	92½	95½	7%

89. The problems in buying and selling stock are very simple when one understands the meanings of all the terms involved. The two most important problems are:

(1) *Stock bought at one price and sold at another is a gain or loss of how much?*

(2) *Stock bought at a certain price and paying a certain dividend yields what per cent on the investment?*

ORAL PROBLEMS.

1. Stock quoted at $135\frac{1}{8}$, if bought through a broker, will cost how much per \$100 share? What will 10 shares cost?

NOTE.—Always consider brokerage $\frac{1}{8}\%$ of the par value.

2. When stock is quoted at $86\frac{3}{8}$, what will a \$100 share cost if bought through a broker? What will a \$50 share cost? What will a \$500 share cost?

3. How much money will I receive for each \$100 share of stock sold through a broker at $163\frac{5}{8}$? How much will I receive for 10 such shares? What will the broker get for his work?

4. If I buy stock at $143\frac{1}{8}$ and sell it at $160\frac{1}{8}$, both transactions through a broker, what will I make on each \$100 share? How much will I make on 10 such shares?

5. If a corporation whose capital stock is \$100,000 divides \$6000 among its stockholders, what is the rate of dividend? What will a man receive who has \$1000 of this stock?

6. \$50,000 of the \$250,000 capital stock of a company is preferred stock, paying 7%. The remainder is common stock. If \$19,500 is divided among the stockholders, what rate of dividend is declared upon the common stock? What does a holder of \$2500 of the common stock receive?

7. If I own 20 \$100-shares of stock in a corporation, and a 10% dividend is declared, how much will I receive?

8. If I own 10 \$1000-shares of stock, and a 5% quarterly dividend is declared, what do I receive each quarter? What do I receive in a year?

9. If I receive \$400 as my 8% dividend from a certain stock, how many \$100 shares have I? What did they cost me, if bought through a broker, when quoted at $119\frac{1}{8}$?

10. If my $4\frac{1}{2}\%$ dividend is \$90, how many \$100 shares have I? What can I realize by selling it through a broker, when quoted at $92\frac{1}{8}$?

11. If I buy stock quoted at $199\frac{1}{8}$, through a broker, and receive a 20% annual dividend, what per cent of my investment is this? How much better is this investment than money loaned at 6% interest?

12. When stock bought through a broker, at $49\frac{1}{8}$, pays a 4% annual dividend, to what rate of interest is it equivalent?

13. What can I afford to pay for stock paying a 12% annual dividend to receive 6% on my investment? If I can afford to pay but \$200 for a \$100-share, at what *market* quotation must I buy through a broker to make this?

14. If my 5% quarterly dividend on some stock is \$250, how many \$100-shares have I?

15. If I buy stock at $119\frac{1}{8}$, through a broker, receive a 15% dividend, and sell in one year for $125\frac{1}{8}$, through a broker, how much have I invested per share, and what have I really gained? What per cent of my investment is my gain? How much better is this than 6% interest?

16. What does one really lose, besides the interest on his money, by buying at $149\frac{1}{4}$, and selling at $80\frac{1}{8}$, after receiving 5 annual 12% dividends? If money is worth 5% interest, what is the total loss?

17. My broker, after deducting his brokerage of $\frac{1}{8}\%$, remitted \$7,412.50 from a sale of 100 shares of stock. At what price did he sell the stock?

18. My broker bought 20 shares of stock for me. The total cost, including his commission, was \$1,640. What was the market price of the stock?

WRITTEN PROBLEMS.

(Use the data in the table on page 83 for Problems 1-15.)

NOTE.—Consider brokerage $\frac{1}{8}\%$ of the par value on each transaction.

1. What would 40 shares of Atchison have cost in 1906? What would it have brought in the market Dec. 24, 1907? What would have been the net loss, had one annual dividend been received?

2. What would C. M. & St. P. yield at the last rate of dividend, if bought in 1906? (That is, what per cent of $199\frac{1}{8} + \frac{1}{8}$

is 7?) What would the same stock yield if bought Jan. 27, 1908?

3. Observing the tables, show why dealing in stocks is more a speculation than an investment.

4. Find the gain in buying 50 shares of Reading on Nov. 23, and selling them March 13.

5. Find the net loss in buying 50 shares of Great Northern in 1906, and selling them, Dec. 24, 1907, one dividend having been received.

6-15. From the daily quotations (found in your newspaper) find the gain or loss from an investment in 50 shares of each of the stocks quoted on page 83 bought, Feb. 22, 1908, and sold at the time you are studying this.

16. Find the per cent of rise or decline, month by month, in any of the following stocks, as your teacher may direct.

Stocks.	Low, 1907	Jan. 3, 1908	Feb. 3, 1908	Mar. 3, 1908	Stocks.	Low, 1907	Jan. 3, 1908	Feb. 3, 1908	Mar. 3, 1908
Am. C. & F. . .	24 $\frac{3}{4}$	31 $\frac{1}{4}$	29 $\frac{1}{4}$	27	B. & O. . . .	75 $\frac{3}{4}$	82 $\frac{3}{4}$	88	79 $\frac{1}{4}$
C. & O. . . .	23 $\frac{1}{4}$	30 $\frac{3}{4}$	29 $\frac{1}{4}$	26 $\frac{1}{4}$	Ch. M. & St. P.	93 $\frac{1}{4}$	105 $\frac{3}{4}$	111 $\frac{1}{4}$	109 $\frac{3}{4}$
Ch. & G. W. . .	6 $\frac{3}{4}$	8	5	4	Gt. N. pf. . .	107 $\frac{1}{4}$	118	120 $\frac{1}{4}$	117 $\frac{1}{4}$
Erie.	12 $\frac{1}{4}$	17	15	13	N. Y. Cent. . .	89	92 $\frac{3}{4}$	96 $\frac{3}{4}$	94 $\frac{1}{4}$
M. K. & T. . . .	20 $\frac{3}{4}$	25 $\frac{1}{4}$	22 $\frac{1}{4}$	18	N. P.	100 $\frac{1}{2}$	120 $\frac{3}{4}$	123 $\frac{1}{4}$	121 $\frac{3}{4}$
Mo. P.	44 $\frac{3}{4}$	45 $\frac{3}{4}$	42	31 $\frac{1}{4}$	Penn.	103 $\frac{1}{4}$	111 $\frac{1}{4}$	111 $\frac{1}{4}$	112 $\frac{3}{4}$
R. I.	11 $\frac{1}{4}$	15 $\frac{1}{4}$	11 $\frac{1}{4}$	11 $\frac{1}{4}$	Reading. . . .	70 $\frac{1}{4}$	99	100 $\frac{1}{4}$	95 $\frac{3}{4}$
U. S. Steel. . .	21 $\frac{1}{4}$	26 $\frac{3}{4}$	28	28 $\frac{1}{4}$	U. S. St. pf. .	79 $\frac{1}{4}$	89	92 $\frac{3}{4}$	92 $\frac{3}{4}$

17. Make the same sort of comparisons with the market prices at the time you are studying this.

18. A man bought 40 shares of L. & N. stock at $156\frac{1}{2}$, and received an annual dividend of 10%. What was the total amount of the investment? What was the annual dividend? To what rate of interest is it equivalent?

19. On May 13, 1906, a man bought 30 shares of Cent. of N. J. at $239\frac{1}{8}$. He received two 8% dividends, and sold Dec. 13, 1907, at $158\frac{1}{8}$. What was the total loss, counting money worth 6% interest?

20. On May 5, 1905, I bought 50 shares of N. Y. Central stock at 143, received an 8% dividend, and sold Jan. 5, 1906, for $156\frac{1}{4}$. What was my net gain, money being worth 5%? If I had not sold until Dec. 5, 1907, I should have received another dividend of 6% and sold for $92\frac{1}{8}$. What would have been my net loss?

21. In 1900, I bought 60 shares of stock at $119\frac{1}{8}$, which paid me three 10% dividends. In 1903, I sold it for $75\frac{1}{8}$ and invested in stock at $59\frac{3}{8}$, which is now (1908) worth $80\frac{1}{8}$, and has paid me four 5% dividends. What has been my average yearly rate of gain or loss on my investment?

22. A man bought 40 shares of C. M. & St. P. stock in 1901 at 162. If he has received a yearly dividend of 12% for 6 years, and is compelled to sell in 1907 at $103\frac{1}{2}$, how much does he lose? (Consider that the investment has run 6 years and that money is worth 5% simple interest.)

90. Bond investments. It has been seen that when an individual borrows money from another individual, or from a bank, the borrower gives the one of whom the money is borrowed a written promise to pay at some fixed time. This is a promissory note. When a corporation, municipality, or government wishes to borrow money, instead of finding a lender and giving a note for the sum borrowed, it issues

interest-bearing notes called *bonds*. These are put on sale, and anyone having money to lend or invest may buy them.

While the denomination of the standard bonds is usually \$500 or \$1000, the *market quotation* is per \$100.

A **bond**, then, is merely a written agreement to pay a certain sum within a certain time, and to pay a fixed rate of interest. The bonds of a stock corporation are secured by a mortgage on the property of the company. The bonds of a city, county, state, or national government, cannot be secured by a mortgage, of course; but since they are paid by a tax, they are considered the safest kind of bonds.

91. A registered bond is a bond registered in the holder's name on the books of the authority issuing it. When the interest is due, it is sent direct to the holder.

92. A coupon bond is a bond that has as many coupons, or dated certificates, attached as there are interest payments to be made. Such coupons, upon maturity, become independent demands, and may be detached from the bonds and collected personally or through a bank, just as any other kind of demand.

93. Bonds as an investment. Bonds are usually bought for the interest they yield and not for a rise in value. Tax bonds, as city, county, state, or government bonds, and also first mortgage railroad or industrial bonds, where the corporation is earning a reasonable per cent of its capitalization, are conservative investments. Such securities, on account of their safety, find a ready sale at a comparatively low rate of interest. However, one dependent upon the income from his investments for his support should select such bonds in preference to stocks.

*The following table shows the stability of bonds and the rates of interest : **

RAILROAD BONDS.

Name of Bond	Range in 1907	Last sale in 1907	Rate of interest	Date of maturity
Atchison E. Okla.	93—91	93	4%	March 1928
Baltimore & Ohio	105 $\frac{7}{8}$ —105 $\frac{7}{8}$	105 $\frac{7}{8}$	5%	Feb. 1919
Chi. Mil. & St. P.	105 $\frac{5}{8}$ —104	104	6%	Jan. 1910
Ill. Cent.	102 $\frac{3}{4}$ —102 $\frac{3}{4}$	102 $\frac{3}{4}$	4%	Dec. 1950
N. Y. Cent.	100 $\frac{3}{4}$ —100 $\frac{3}{4}$	100 $\frac{3}{4}$	4%	Jan. 1940
Northern Pac.	95 $\frac{3}{4}$ —95 $\frac{3}{4}$	95 $\frac{3}{4}$	4%	Dec. 1996

ORAL PROBLEMS.

1. When bonds are quoted at 97 $\frac{3}{8}$, what will a \$1000 bond cost, if bought through a broker? What is it worth when due?

2. A corporation is bonded for \$200,000. The bonds bear 4 $\frac{1}{2}$ % interest. What yearly interest does the corporation pay?

3. What income will one receive yearly from six \$500 5% bonds?

4. A \$30 coupon was detached from a 6% bond, being the interest for one year. What was the face of the bond?

5. How many \$1000 4% bonds are necessary to yield an annual income of \$1200?

6. When one buys 5 per cent 3-year bonds at 96 $\frac{7}{8}$, brokerage $\frac{1}{8}$ %, what does he make besides the interest if he holds them until maturity?

NOTE.—Notice that 3-year bonds costing 97 mature in 3 years at \$3 on the \$100 more than they cost. This is an average of \$1 per year on each

* It must not be thought from this selected list of 1st mortgage railroad bonds, that *all* bonds are equally stable. The earnings of a corporation may be such that even their bonds are not desirable securities, and the price falls. Industrial bonds usually fluctuate more than railroad or tax bonds.

\$100 besides the \$5 interest. Hence, from an investment of \$97 the average yearly income is \$6. These bonds then, yield $6\frac{1}{3}\%$ annually. In practice this would more likely be expressed as "nearly 6.186%," instead of by the common fraction.

This method is not quite accurate, but it gives the *approximate yield*. Why?

7. Certain 3-year 5% bonds, in which the makers of the bonds held the option of maturing them at 101 in 2 years, were advertised by a broker at 97. The broker explained that these bonds held to maturity would yield over 6%, and if matured in 2 years, would yield over 7%. Explain why this is true.

8. What is the average yearly income per \$100, on 4% 10-year bonds bought at $79\frac{1}{8}$ and brokerage? To what rate of interest is this equivalent?

9. Which would you prefer, 5% bonds bought at 90, to mature in 5 years, or bought at 80 to mature in 20 years?

WRITTEN PROBLEMS.

NOTE.—Consider brokerage $\frac{1}{8}$ of the par value in each transaction.

1. In Jan., 1908, American Tel. & Teleg. Co. 5s, due in 1910, sold for $92\frac{1}{2}$. What rate of interest will they yield, if held till maturity?

SUGGESTION. These bonds cost, including brokerage, \$92.625 per \$100. In 2 years they mature for \$7.375 per \$100 more than they cost. This makes an approximate yearly income of \$8.6875 on an investment of \$92.625.

2. In Feb., 1908, Mich. Cent. 5s, due in Feb. 1910, sold at 96. What rate of interest will they yield, if bought and held till maturity?

3. Lake Shore & Mich. Southern 4s, due in 1931, could be bought in 1908 for 85. At this price, what would they yield the investor?

NOTE.—The method of Problem 1 gives a yearly average of \$0.6467 from the increased value of the bonds at maturity. Since the *time to run* is so long, 23 years, this is much too large, *i. e.*, this sum deposited yearly at even a low rate of interest, compounded annually, will amount to much more than \$14.875.

\$1 deposited annually, at 4% compound interest for 23 years, will amount to \$38.08 (see § 94). Hence, $\$14.875 \div 38.08$, or \$0.3906, yearly for 23 yrs., will amount to the increased value of the bonds at maturity. Hence, the maturity value adds but \$0.3906 yearly to the regular interest. This is the method used when the *exact* yield is required.

When the time is short, the first method gives a close *approximate* yield.

4. Which is better and how much, D. & H. 4s at $92\frac{3}{8}$ or C. & N. W. 6s at $114\frac{1}{8}$, brokerage $\frac{1}{8}$, if both are due in 10 years?

NOTE.—Make use of compound interest at 4% and find the *exact* yield.

5. Milwaukee Electric Ry. $4\frac{1}{2}\%$ bonds, due Jan., 1931, subject to call at 108 after Jan., 1916, can be bought in Jan., 1903 at 95. What is the approximate yield if held to maturity? What is the approximate yield if called Jan., 1916, at 108?

6. In 1908 a man bought, through a broker, the following bonds:

A \$1000 New York City $4\frac{1}{2}\%$ bond, due in 1917 at $102\frac{1}{8}$;

A \$1000 Milwaukee $3\frac{1}{2}\%$ bond, due in 1921 at $92\frac{1}{4}$; and

A \$500 Rock Island 4% bond, due in 1919 at $96\frac{1}{8}$.

Find the total investment; the total yearly income; the approximate yield of each bond if held to maturity.

7. In 1904 C. R. I. & P. 4s sold for 105. In 1908 they dropped to 87. Find the net loss of a man who bought two \$1000 bonds in 1904 and sold them in 1908, not regarding interest on his money. Find the net loss considering money worth 4% compound interest.

94. Accumulative bonds. The accumulative bond method of investment offers an opportunity for one to invest small regular incomes in order to secure a larger sum. These bonds *mature*, or become due, when a series of equal regular payments amount, with a fixed rate of interest compounded regularly, to the face of the bond.

Thus, a \$1000 10-year 6% accumulative bond is sold for a regular yearly (or any other period agreed upon) payment. The sum of these ten (in case

they are yearly) payments, together with the compound interest at 6% that has accrued upon each at the time of settlement, amounts to \$1000 at the end of 10 years. The bond then has *matured* or becomes *due*.

NOTE.—Instead of using the compound interest tables as in Ex. 1, § 79, time is saved in the problems of this section, by using the following table which shows the sum to which \$1, paid at the beginning of each year, will increase in any number of years from 1 to 40.

Yr	3%	3½%	4%	5%	6%	Yr
1	1.0800	1.0850	1.0400	1.0500	1.0800	1
2	2.0909	2.1062	2.1216	2.1525	2.1836	2
3	3.1836	3.2149	3.2405	3.3101	3.3746	3
4	4.3091	4.3625	4.4103	4.5256	4.6371	4
5	5.4684	5.5502	5.6330	5.8019	5.9753	5
6	6.6625	6.7791	6.8983	7.1420	7.3938	6
7	7.8923	8.0517	8.2142	8.5491	8.8975	7
8	9.1591	9.3685	9.5828	10.0266	10.4913	8
9	10.4639	10.7314	11.0061	11.5779	12.1808	9
10	11.8078	12.1420	12.4864	13.2068	13.9716	10
11	13.1920	13.6020	14.0258	14.9171	15.8699	11
12	14.6178	15.1130	15.6268	16.7180	17.8826	12
13	16.0863	16.6770	17.2919	18.5986	20.0151	13
14	17.5989	18.2057	19.0236	20.5786	22.2760	14
15	19.1569	19.9710	20.8245	22.6575	24.6705	15
16	20.7616	21.7050	22.6975	24.8404	27.2129	16
17	22.4144	23.4997	24.6454	27.1324	29.9057	17
18	24.1169	25.3578	26.6712	29.5390	32.7600	18
19	25.8704	27.2797	28.7781	32.0660	35.7856	19
20	27.6765	29.2695	30.9692	34.7193	38.9927	20
21	29.5368	31.3290	33.2480	37.5052	42.3923	21
22	31.4529	33.4604	35.6179	40.4305	45.9958	22
23	33.4265	35.6665	38.0826	43.5020	49.8156	23
24	35.4593	37.9499	40.6459	46.7271	53.8645	24
25	37.5530	40.3191	43.3117	50.1135	58.1564	25
26	39.7096	42.7591	46.0842	53.6661	62.7058	26
27	41.9309	45.2906	48.9676	57.4026	67.5281	27
28	44.2188	47.9108	51.9663	61.3227	72.6398	28
29	46.5754	50.6227	55.0849	65.4388	78.0582	29
30	49.0027	53.4295	58.3283	69.7608	83.8017	30
31	51.5028	56.3345	61.7015	74.2988	89.8898	31
32	54.0778	59.3412	65.2095	79.0638	96.3432	32
33	56.7302	62.4532	68.8579	84.0670	103.1838	33
34	59.4621	65.6740	72.6522	89.3303	110.4348	34
35	62.2719	69.0076	76.5983	94.8363	118.1209	35
36	65.1742	72.4579	80.7022	100.6881	126.2681	36
37	68.1594	76.0289	84.9708	106.7995	134.9042	37
38	71.2342	79.7219	89.4091	113.0850	144.0585	38
39	74.4013	83.5503	94.0255	119.7998	153.7620	39
40	77.6633	87.5095	98.8265	126.8398	164.0477	40

PROBLEMS.

1. The American Real Estate Co. of New York sells a \$1000 10-year 6% accumulative bond for \$71.57 yearly. Will this sum yearly amount to \$1000 in 10 years at 6% compounded yearly?

2. A \$1000 20-year accumulative bond sells for \$25.65 yearly. Is this 6% compound interest?

3. What could you afford to pay yearly for a \$4000 10-year 5% accumulative

bond, interest compounded annually?

4. What could you afford to pay yearly for a \$5000 15-year 6% accumulative bond, interest to be compounded annually?

5. Find the semi-annual payments that a corporation must charge upon a \$1000 10-year 6% accumulative bond, interest to be compounded semi-annually.

6. A man 40 years old can take out a \$1000 15-year endowment insurance policy for \$63.23, payable annually. This same payment made upon a 6% accumulative bond would amount to what in the same time?

7. If a man can begin investing \$500 annually at 6% when 25 years of age and continue it until he is 60, keeping all interest re-invested as it accrues, what will he have accumulated? Find the annual income from this accumulation if invested in 4% bonds at 98.

8. Suppose that one invests \$200 yearly at 6% from age 20 to 30; \$500 annually from 30 to 40; and \$800 annually from 40 to 50. Keeping all interest re-invested what will he have accumulated at 50? What annual income may be received from this if invested in 5% bonds at 102?

95. Building and loan associations. The primary object of a building and loan association is to afford its members a means of investing their savings weekly or monthly at a fair rate of interest, and to loan its members funds for building purposes that may be repaid by weekly or monthly installments.

Such associations usually issue *stock* in \$100 shares. These shares are said to *mature* when the weekly or monthly payments, called *dues*, together with the earnings, usually called *dividends*, are equal to \$100.

I. *The earnings* of the association largely consist of the *interest* from loans made to its members. In some associations, however, one who wishes to borrow a sum of money offers a *premium* in order to get it, since there may be a greater demand.

for a loan than there is cash on hand. Again, there are *fines* for failure to pay the dues at the proper time.

There is also a gain from another source to those who keep up their payments until their stock matures; viz: one withdrawing his stock before it matures usually receives back the payments made, with interest upon the same for a rate less than the real earnings.

In general, then, the gains of the association are from (1) *Interest on loans*; (2) *Premiums paid for loans*; (3) *Fines for delinquency in payment of dues*; (4) *Profits from withdrawals*.

II. *The dividends* or earnings are credited to the stockholders at regular periods, usually twice a year. These dividends are usually reckoned at a rate of interest, say 6% or 7%, depending upon the earnings, on the various amounts paid in.

Thus, if a man paying \$10 monthly had a credit of \$156 when receiving his last credit of dividends, and has paid in \$10 monthly for 6 months, he receives interest on \$156 for 6 months, \$10 for 5 months, \$10 for 4 months, and so on. The interest for these amounts at 6% would be respectively \$4.68, \$0.25, \$0.20, \$0.15, \$0.10 and \$0.05, or a total of \$5.43, now giving him a credit of \$221.43 toward maturing his stock.

PROBLEMS.

1. If a man wishes to borrow \$4000 from a building and loan association, he takes \$4000 of its stock and pays dues until the stock matures, and this cancels the indebtedness. If he pays \$40 per month, part of this pays the interest at 6% on the \$4000 that he owes the association and the remainder is used as dues to mature the stock. How much of the \$40 is needed to pay 6% interest on \$4000? How much is left as monthly dues? Find how long it will take to mature the stock if the earnings are sufficient to pay 7% interest, credited semi-annually. Suppose that he made his first payment upon an interest or dividend paying date.

SUGGESTION. \$20 of the \$40-payment are *dues*. When the first interest period occurs he should have had \$20 drawing interest for 6 months, the same for 5 months, and so on to \$20 for 1 month. This amounts to \$122.45, which, together with his monthly payment, gives \$142.45 that draws interest for 6 months before the next interest period. Compute, in this way, until the amount equals \$4000 and count the time that elapses.

2. Suppose that a man, after paying \$25 monthly for 5 years, withdraws from a B. & L. association and receives all money paid in, together with 6% interest. What does the association gain from this withdrawal, the dividends being 7%, credited semi-annually?

3. Instead of borrowing money with which to build a home, a man matures 20 \$100 shares of B. & L. stock by paying \$20 monthly. How long will it take this stock to mature if the dividends are 8%, credited semi-annually?

4. How long will it take for a \$25 monthly payment to mature 50 \$100 shares at a 7% dividend credited semi-annually?

96. Life insurance. Life insurance, and especially *endowment insurance*, may be considered a form of investment. A **life insurance policy** is a contract whereby one party for a stipulated sum, called a **premium**, agrees to pay a given sum or sums to the heirs of the second party (the insured) or to some other person or persons designated by the second party, at his death.

97. Endowment insurance is a form of life insurance wherein the insurer agrees to pay the insured the whole amount named in the policy if he survives beyond a specified date, or to pay it to his heirs or to any other person designated if he dies before this date.

98. The elements of a premium. The annual **net** or **mathematical premium** is a sum which, if annually invested at a fixed rate of compound interest, will amount to a sum sufficient to

pay all obligations of the contract. This amount is determined by a given *mortality table*. The **gross premium**, or the premium actually collected by the insurance company, is the net premium plus a fixed per cent of the net premium which has been added to meet the expenses of the management. This part which is added to the net premium is called the **expense loading**.

99. Mortality tables. A mortality table shows the number of persons out of a certain number of a given age that die during the year and during each succeeding year thereafter. These tables are made up from actual experience extending over a long period of years. The *American Mortality Tables* are compiled from the experience of the New York Mutual Life Insurance Company.

100. The reserve. The annual premium at any age is not only a sum to meet the expenses of the management and the death claims, called *mortality cost*, of the current year (computed according to mortality tables) but also a sum which, from this and succeeding premiums, will, at a fixed rate of compound interest (usually 3% or $3\frac{1}{2}\%$), mature the policy in a given time. This is called the **reserve**. The reserve, then, matures the policy just as the payments on an accumulative bond or the dues of a building and loan association mature the bond or the stock.

It is evident that the rate of mortality cost at any given age is the same whatever kind of policy is considered. Then an endowment policy must charge a sum in excess of the mortality cost and expense loading that will mature the policy in the given time at the given rate of interest.

101. The surrender value of a policy. It has been seen that the reserve is a part of each premium that goes to create a fund which in a given time amounts to the face of the policy.

The reserve, then, at any time, really belongs to the policy-holder. The company will, upon the surrender of the policy (their contract), give the holder a cash payment which is practically the reserve on the policy. This is called the *cash surrender value* of the policy.

102. The surplus. There are three main sources of surplus in a life insurance company, viz: (1) *A saving from the expense loading in a premium*, for most companies provide for a greater expense than is actually used. (2) *Saving in mortality cost*, for by using care in selecting risks, the *actual* mortality is usually less than the *expected* mortality upon which the premiums were based. (3) *The interest earned on the reserve fund in excess of the rate assumed in computing it*, for just as a savings bank that pays 3% or 3½% upon the deposits receives a rate in excess of this, so a life insurance company usually earns more than the rate upon which the reserve element of the premium was computed.

103. The dividends. That part of the surplus which is distributed among the policy-holders is called the *dividend*. It is reckoned as a per cent of the total reserve of the company. Each policy-holder, then, receives this per cent of the reserve that is credited to his policy.

Those policies whose policy-holders receive a dividend are called **participating-policies**. All others are **non-participating**. Since the holders of non-participating policies do not receive a dividend, they pay a smaller premium.

104. The problems of insurance. The computation of payments, etc., upon the various policies is done by an *actuary*. Such computations are too difficult for elementary mathematics. There are problems, however, that arise in the minds of the buyers of insurance that should be understood by every one.

PROBLEMS.

1. A life insurance company has a membership of 81,822, each insured for \$1000. Premiums are collected at the beginning of the year. The company expects that, during the year, 732 members will die, and their claims must be paid at the end of the year. If the company earns 4% on the premiums collected, how much must it charge each of its members at the beginning of the year in order to meet its losses?

2. The *net annual* premium of an ordinary life policy at age 20, on a 3% reserve, based upon the American Mortality Tables, is \$14.41 per \$1000. What premium does a company collect that has a 25% expense loading? What is the annual premium in such a company upon a \$7500 policy?

3. On a $3\frac{1}{2}\%$ reserve basis, the *net annual* premium per \$1000, age 30, is \$17.19. A certain company sells a non-participating policy at this age for \$20.20. What is their per cent of expense loading? To what sum will the loading amount in 40 years at $3\frac{1}{2}\%$ compound interest, *i. e.*, if the man lives until he is 70, what has he contributed to the expenses of the company?

4. On a $3\frac{1}{2}\%$ reserve basis, the net premium per \$1000, age 25, is \$15.10. With a 25% expense loading, what will a company collect yearly on a \$5000 policy?

5. The gross premium of a certain insurance company, at age 35, is \$26.35. The net premium, on a 3% reserve basis, is \$21.08. This company spends 86% of its loadings. What is its per cent of expense loading? What per cent of its gross premiums are its actual expenses?

6. The reserve at the end of 1 year from a payment of \$20.14, age 25, 3% reserve, loading 25%, is \$8.60. How much of the premium was estimated for mortality cost?

SUGGESTION.—The premium is paid in advance. Hence the \$8.60 is the amount of the reserve at 3% for 1 year.

7. Mr. Allen, at the age of 30, insured his life for \$1000 on the ordinary Life plan, premium \$21.95. Mr. Brown, also age 30, deposited \$21.95 annually in the Savings Bank at 3% compound interest. Both died at age 38. Which investment proved the more profitable? How much was the difference?

Make a comparison if both die at age 48. At age 58. At age 68.

8. At age 42, a man took out an \$8000, 20-payment, non-participating policy at \$37.20 per \$1000. If he dies at 64 (22 years after making the first payment), compare the amount left his estate with that of the amount of these premiums at 5% interest compounded annually.

9. In 1892, a man took out a \$10,000, 15-year endowment policy which cost him \$56 per \$1000 yearly. To what would his total payments have amounted at 3% compounded annually? How much more would he have had in 1907 when the policy was paid, had the money been in the bank drawing 3% interest compounded annually? What then did his protection really cost him?

10. If a man, age 45, takes out a \$10,000 ordinary life policy for \$33.32 per \$1000, and dies after having made 11 payments, compare the amount left his estate with the amount of these payments at 5% interest compounded annually. (Suppose that he dies $10\frac{1}{2}$ years after making the first payment.)

11. A man, age 35, took out a \$1000, 25-year endowment, annual premium \$39.97 (net premium \$33.18 + \$6.79 loading). At the beginning of the 10th year he has a reserve credit of \$297.50. How much of his 10th premium is credited to reserve? The mortality cost for the current year is \$10.83 per \$1000.

SUGGESTION. Since the reserve lessens the actual risk of the company, their risk, at this time, is but \$1000—\$297.50.

12. In Example 11, the insured had a reserve credit of \$297.50

at the beginning of the 10th year, and it was found that \$25.57 was to be credited from his 10th premium. At 3% interest, what is his reserve credit at the end of the 10th year? What should be the surrender value of his policy at that time?

13. An \$8000, 15-year endowment policy, at age 40, costs annually \$63.23 per \$1000. The annual premium would buy a 5% 15-year accumulative bond of what size?

14. A man insured for \$1000 at age 30 on the 20-year endowment plan, paying an annual premium of \$43.07. At the end of 20 years he was paid \$1000 in cash. If he had deposited the amount of his premium annually for twenty years in the savings bank at 3% compound interest, how much would he have had to his credit? On that basis, what was the expense to him for the risk of insurance?

15. A man, age 41, took a \$10,000 ordinary life participating policy at \$32.02 per \$1000, the yearly dividends to be applied to his premiums. At the end of the 3rd year his reserve credit is \$409.40. If the company declares a 4% dividend, how much will it reduce his 4th premium?

If his reserve at the end of the 10th year is \$1790 and a 4% dividend is declared, what is his 11th premium?

If his reserve at the end of the 15th year is \$2897 and a 5% dividend is declared, what is his 16th premium?

16. A man, age 38, took out a \$15,000 ordinary life policy, annual premium \$25.78 per \$1000. If he dies after making the 12th payment, how much more has he left his estate than he would have left it by investing the premiums at 4% interest compounded annually?

17. A man, age 40, can take non-participating ordinary life for \$27.62 annually per \$1000. If he takes a \$15,000 policy and dies at 55 (having paid 15 premiums), how much more does he leave his estate than he would have left it by putting these

payments into a savings bank at 4%, credited annually? Compare the amount left if he dies at 65 with the amount of the premiums at 4% compounded annually. Compare the two amounts if he lives to be 75.

18. A man, at age 39, may carry ordinary non-participating life for \$26.68 per \$1000, or 10-payment life for \$53.57. At the end of 10 years the surrender value of the first is \$160.25 per \$1000, that of the other \$415.30. He wishes to know whether to take \$2000 of the first or \$1000 of the second. If he has to surrender his policy at the end of 10 years, show the actual cost of the extra protection if he takes the first.

CHAPTER IV.

CANCELLING INDEBTEDNESS.

105. Paying at a distance. If one wishes to pay a bill in some other city, he may express the money to the party to whom the bill is payable ; he may put the money in an ordinary package and mail it ; or he may send it by registered mail. These methods, however, are not the common ways of cancelling indebtedness.

106. Exchange. Debts are more often cancelled without sending the actual money, but by sending *postal or express money orders, checks, or drafts*. This method is called *exchange*.

107. Postal or express money orders. The postal service and some express companies keep large sums in many offices. A postmaster or express agent can write an order directing the postmaster or express agent at another office to pay a sum of money to any person you may name. For this, you pay the sum named in the order together with a small fee of from 3c. to 30c., no matter what the distance ; or, if you send to a foreign country the fee is from 10c. to \$1. This fee is the *cost of exchange*.

NOTE. Post-office orders are payable at an office named ; express orders at any office of the same company.

The charges for postal or express money orders payable in the United States are as follows:

Not over \$2.50	8c	Over \$30.00 to \$40.00	15c
Over \$2.50 to 5.00	5c	Over 40.00 to 50.00	18c
Over 5.00 to 10.00	8c	Over 50.00 to 60.00	20c
Over 10.00 to 20.00	10c	Over 60.00 to 75.00	25c
Over 20.00 to 30.00	12c	Over 75.00 to 100.00	30c

NOTE. The maximum amount for which a single postal money order may be issued is \$100.

108. Checks. One having money on deposit in a bank may issue an order upon the bank to pay a sum (not exceeding the deposit) to any person he may indicate. This order is called a **check**. The following is a common form :

YPSILANTI, Mich., <i>June 4, 1908.</i> No. <i>1243.</i>	
THE FIRST NATIONAL BANK OF YPSILANTI	
Pay to <i>Simmons & Newton</i> or order.....	<i>\$98.20</i>
<i>Ninety-eight and $\frac{20}{100}$</i>	Dollars.
<i>E. A. Bacon.</i>	

Banks usually charge a small fee, called *exchange*, for collecting a personal check upon a bank in some other city. For this reason it is hardly considered business courtesy to send a personal check to cancel indebtedness. Checks are usually used only in one's own city.

109. Drafts. A bank keeps a deposit in some bank, called a **correspondence bank**, in one or more of the large commercial centers. To cancel a debt in some other city, one usually gets his bank to issue an order on its correspondence bank to pay him a certain amount. This order is called a **draft**. A draft, then, is like a check except that it is the order of the cashier of a bank drawn on some other bank. A draft will be taken by a

bank anywhere at *par*. That is, a bank makes no charge for collection, but pays the bearer the full amount of the order. The usual form of a draft is as follows:

<i>\$234.</i>	ALBANY, N. Y., <i>July 8, 1908.</i>
TENTH NATIONAL BANK	
Pay to the order of <i>Howe & Co.</i>	
<i>Two hundred thirty-four and</i> $\frac{no}{100}$ Dollars.	
To the Farmers' National Bank, Syracuse, N. Y.	
<i>Pu'rick Matthews,</i> Cashier.	

110. Cost of a draft. When selling drafts, banks are ordering paid out the money they have on deposit in the banks of these large commercial centers. This would soon exhaust their balances at these banks and thus require the large expense of frequent transfers of large amounts of actual money, were it not for the fact that they are also constantly buying drafts on other banks, and thus paying out money for drafts which they mail to the banks upon which they issue drafts. In this way they keep up their deposits without often having to actually send money when the accounts between these banks are regularly balanced. As there must always be a balance in favor of the bank upon which the drafts are drawn, the loss of interest and the necessary office expenses make it necessary for banks to charge a small fee above the actual face of the draft, say 10 cents for sums less than \$100, 25 cents for sums from \$100 to \$500, etc. This is called a *premium*. The premium upon large drafts is usually some small per cent of the face of the draft.

NOTE.—The rate of exchange is not at all uniform among banks.

111. Indorsements. In buying a draft, one usually has it made out to himself, then, across the back of it, he writes an order for it to be paid to the party that he wishes to have collect the money.

Thus, if D. E. Cole wishes to pay a sum to Robt. Smith, he will buy a draft payable to D. E. Cole, then across the back, write "pay to Robt. Smith, or order, D. E. Cole." This is called an *Indorsement*. This is an "indorsement in full." If Mr. Cole merely signs his name, or "indorses in blank," any one having the draft in his possession may collect it. If Mr. Cole wishes, he may have the draft made out directly to Robt. Smith, in which case, of course, it will need no indorsement. The former method, however, is the usual one, for in effect it is a receipt for the payment for it shows of whom Robt. Smith received the payment.

112. Commercial drafts. Instead of the debtor sending the creditor a draft, as in the last article, the creditor may draw upon the debtor directly by a draft. This form of draft is called a **commercial draft**.

Thus, suppose Haines & Co., of Topeka, have bought goods of S. T. Richards & Co., of Chicago, and have not paid the bill when due. say 30 or 60 days after the goods were bought. Richards & Co. may make out a draft as follows and deposit it with their bank in Chicago for collection.

No. 3465.	CHICAGO, Ill., May 6, 1908.
At sight pay to the order of	
<i>The State Bank of Chicago</i>	<i>\$348</i>
<i>Three Hundred Forty-eight and $\frac{no}{100}$</i>	Dollars.
To <i>Haines & Co.,</i>	
<i>Topeka, Kas.</i>	<i>S. T. Richards & Co.</i>

The Chicago bank now sends this draft to some bank in Topeka. The Topeka bank sends a messenger to Haines & Co. and presents the draft for

payment. If collected, the Topeka bank sends the money, or its equivalent, to the State Bank of Chicago. This bank notifies S. T. Richards & Co. and places the amount, less a small fee for collection, to their credit.

In case Haines & Co. refuse to pay it, the draft is returned to the State Bank of Chicago and Richards & Co. are notified. They must now take other means of collecting.

I. *Sight and time drafts.* The form shown above is that of a **sight draft**. If "Thirty days after date," or some other time is written in place of "At sight," the draft becomes a **time draft**. It is the custom of some houses to sell goods to be paid in 30, 60, or 90 days and at once deposit with their bank a time draft for the payment. To draw upon one by a draft does not imply that this is a means of forcing collection but it is merely a convenience.

Had a time draft been presented to Haines & Co., they would have either written across the face, "accepted," with the date, and the firm's signature, or they would have refused to pay it.

When a time draft is accepted it then has the same force as a promissory note owned by the payee and may be discounted at a bank as any promissory note.

II. *An arrival draft.* Some merchants ship their goods subject to their own order. The shipper receives a **bill of lading** (that is, an acknowledgment of the receipt of goods for transportation) from the freight agent of the transportation company that is to carry the goods. This, properly endorsed, becomes an order upon the agent at their destination to deliver the goods to the party named. This bill of lading is attached to an **arrival draft** and deposited by the shipper at his bank. This bank, through its *correspondence* bank, sends the draft to a bank in the city to which the goods are sent. Upon the arrival of the goods, the one who has bought them is notified and pays the draft and receives the bill of lading, which must be presented before the transportation company will deliver the goods.

The following forms show an invoice sent to the buyer and the draft which is sent with the bill of lading.

INVOICE.

Ypsilanti, Mich., March 24, 1908.

E. B. HODGES & Co.,
Norfolk, Va.

BOUGHT OF MARTIN DAWSON

Via M. C. & N. & W.

To Norfolk, Va.

Hay, Grain and Seeds.

Car M. C. No. 42829.

164 bales, 22445 lb. No. 1 Timothy at \$11.75				
f. o. b. Ypsilanti	131	86		
Arrival Draft			131	86

DRAFT.

\$131.86	Ypsilanti, Mich., March 24, 1908.
On arrival M. C. & N. & W. car No. 42829 pay to the	
Order of Ypsilanti Savings Bank.....	
One hundred thirty-one and $\frac{86}{100}$Dollars	
Value received, and charge the same to account of	
To E. B. Hodges & Co.,	
Norfolk, Va.	Martin Dawson

PROBLEMS.

1. In the check on page 103, who receive the \$98.20?
2. Who pays it to them? Whose money is the bank paying out?
3. What indorsement will be necessary before the bank pays the check?

4. In the draft on page 104, tell of what bank Matthews is cashier.

5. Where is the draft payable? To whom?

6. In what way may Howe & Co. make it payable to another party?

7. If Howe & Co. are going to mail this draft to E. L. Harris & Co., how should it be indorsed? Why is it better for Howe & Co. to "indorse in full," than merely to sign their name across the back?

8. If Howe & Co. prefer, they may have the draft made payable directly to E. L. Harris & Co. Which do you think would be the better plan?

9. Suppose the draft on page 105 to have been a 60-day time draft and to have been accepted May 10. What is the day of maturity and how long has the draft to run before due?

10. If sold on the day of acceptance at 6% discount, what are the proceeds? (Remember that when accepted it is practically a promissory note.)

11. The bank at Topeka will likely charge a small fee for collecting the draft. If they charge $\frac{1}{4}\%$ (of the face of the draft) for collecting (exchange) and discount it on the day it is bought, what shall they remit the State Bank at Chicago?

12. F. Alton of New Orleans draws at sight on F. Fay of Waco, Tex., Aug. 3, 1907, for \$500. Make the draft.

13. Suppose the debt not due till Nov. 1. Make a proper time draft dated Aug. 3.

14. If discounted Aug. 18, the proceeds would be what?

15. If discounted Aug. 3, less an additional $\frac{1}{4}\%$ for exchange, the proceeds would be what?

16. A Charleston bank buys a draft on Richmond for \$2100, charging $\frac{1}{2}\%$ for exchange. If it had taken the draft for col-

lection only, the charge would have been 25ct. To the maker what is the difference in money? In which case is payment made more quickly?

17. Write a draft of which you are the maker, your teacher the payee, and a neighboring bank the drawee.

18. Suppose that you bought of E. L. Holman & Co. a bill of hardware as follows:

26 doz. axes at \$4.80; 32 doz. files at \$1.20; 36 doz. saws at \$3.40. Discounts 40% and 10%. Terms: 2% discount for cash in 10 days; net 60 days.

Make out a bill for this, showing the net cash price. Send a draft in payment. What is the face of the draft? Where will you get it? To whom will you have it payable? If made payable to yourself, how will you indorse it before sending it?

19. F. R. Smith & Sons bought of J. L. Morris & Co. the following:

2 safes at \$12.50; 3 arm-chairs at \$9.00; 2 rockers, leather, at \$25; 1 couch at \$8; 3 couches at \$6.50; 2 rockers at \$9. Terms, net 60 days.

Make out the proper bill finding the price in 60 days. Pay with a 60-day note without interest. Suppose that Morris & Co. receive the note and discount it 5 days after date. What are the proceeds? Who gets it? Who gets the discount? What indorsement is necessary before selling the note? What obligation is assumed when one indorses a note?

20. Suppose you bought of M. E. Dawson & Co. the following:

4 kitchen cabinets at \$13.25; 3 kitchen cabinets at \$15.25; and 1 kitchen cabinet at \$17.75. Terms: 2% off for cash in 10 days; net 30 days.

Make out the proper bill. Find the net cash price. Send them your check for the payment. What will Dawson & Co. do

with this check when they receive it? What will a postal money order to settle the bill cost?

21. A. L. Morgan of Salem, Ill., bought of Reed, Murdock & Co., Chicago, the following:

86 lb. tea at 43 ct.; 280 lb. coffee at 28 ct.; 250 lb. prunes at $11\frac{1}{2}$ ct.; 116 lb. raisins at 14 ct.; 144 cans salmon at 15 ct.; 86 lb. apricots at 18 ct. Terms: 60 days net; 2% off for cash in 10 days.

Make out the proper bill if not paid for 60 days. At the end of 60 days the firm draws a sight draft on Morgan. Make out the proper form. What would Reed, Murdock & Co. do with this draft? What then becomes of it?

22. R. L. Stevens & Sons sold to J. H. Boyce the following on 90 days time, drawing a 90-day draft on Boyce when the goods are billed:

2 rockers at \$13.50; 1 rocker at \$8.25; 2 Morris chairs at \$15.00; 3 chairs at \$3.75; 1 hall tree at \$11.50; 1 hall tree at \$13.75. Terms, net 90 days.

Make out the proper bill; the proper draft. The draft is "accepted" and discounted at 6% in 5 days from date. How much do Steven & Sons really get in cash for the goods?

23. I. C. Carpenter bought of The Van Cleve Glass Co. the following:

3 boxes 7×9 , single, at \$26.75; 5 boxes 10×14 , single, at \$28.25; 2 boxes 16×20 , single, at \$30.00; 5 boxes 12×20 , single, at \$28.00; 6 boxes 18×24 , single, at \$31.75. Discounts 85% and 10%. Terms: 60 days net or 2% off in 10 days.

Write out a check for the cash payment. How much more would the Glass Co. receive by taking a 60-day note without interest and discounting at a bank at 6%, 10 days from the date of the bill?

PART III

ADVANCED PROCESSES, WITH APPLICATIONS TO MENSURATION AND SCIENCE

CHAPTER I.

APPROXIMATE NUMBERS. ABRIDGED PROCESSES.

113. Approximate numbers. The numbers resulting from measurements are, in general, not exact integral numbers, nor even exact fractions, but **approximations**. Common fractions whose denominators have any factor except 2's or 5's cannot be expressed as terminating decimal fractions, but we may express the value decimally to any degree of accuracy that we wish; that is, to tenths, hundredths, etc. A root of a number, too, cannot, in general, be definitely found, but we may get as close an approximation as we wish.

Thus, $\frac{1}{4}=.166\dots$, $\frac{1}{3}=.285\dots$, $\sqrt{2}=1.414\dots$, and $\sqrt{3}=1.732\dots$. These are numbers that never terminate, but they may be carried out to any desired degree of accuracy.

In most of the problems of Part III of this book, we are dealing with *approximations* instead of exact numbers.

114. Operations with approximate numbers. In working with approximate numbers, the results can seldom be depended upon to a degree of accuracy equal to that of the numbers involved. Hence, *in solving a practical problem, it is a waste of time to carry out work to a denomination smaller than that in which the data is expressed.*

Thus, if the approximate edge of a cube is 8.1 in., when the .1 merely denotes that the edge is nearer 8.1 than it is to 8 or 8.2, the volume is not $8.1 \times 8.1 \times 8.1$, or 531.441, a number expressed to the third decimal. This is shown by marking the numbers that are in doubt.

WORK.	EXPLANATION.
8.1	Since .1 is but an approximate
8.1	number, and may stand for .05 or nearly .15, we are
<u>8.1</u>	manifestly in doubt about each number in full-faced
648	type. In fact, we are not sure of the correctness of
<u>65.6 1</u>	the next figure to the left of each figure so marked.
8.1	(Why ?) In this case, then, it is <i>absolutely wrong</i> to
<u>6561</u>	express the answer to the third decimal place. In fact,
52488	the 1 in one's place is in doubt, and the nearest approx-
<u>531.441</u>	imation we can give safely is 530.

115. Abridged processes. If the data upon which a problem is founded is given in approximate numbers, or if for any other reason the result is wanted merely to a certain approximation, the work of multiplication and division may generally be saved by an abridgment. This is best shown by examples.

EXAMPLE 1. Find the product of 84.265 by 3.1416 to the second decimal place.

ABRIDGED WORK.	COMPLETE WORK.
84.265	84.265
3.1416	<u>3.1416</u>
<u>102.795</u>	205590
8.4265	84265
1.3706	187060
842	84265
205	102795
<u>107.65</u>	<u>107.6469240</u>

EXPLANATION. In the abridged work, it is usually more convenient to multiply by the highest number of the multiplier first. Since the result is to be to the nearest 2nd decimal, four decimals are kept in the partial products in order to get accurately the number to "carry" to the column whose sum is desired. If great care is used in expressing the right hand figure of each partial product, but one more place than the number wanted in the complete product is needed.

APPROXIMATE NUMBERS—ABRIDGED PROCESSES 113

EXAMPLE 2. Find the quotient of 26.843 divided by 3.1416, to the second decimal place.

$$\begin{array}{r} \text{ABRIDGED WORK.} \\ 8.54 \\ 3.1416 \overline{) 26.843} \\ \underline{25.18} \\ 171 \\ \underline{157} \\ 14 \\ \underline{18} \end{array}$$

$$\begin{array}{r} \text{COMPLETE WORK.} \\ 8.54 \\ 3.1416 \overline{) 26.8430} \\ \underline{25.1328} \\ 171020 \\ \underline{157080} \\ 139400 \\ \underline{125664} \\ 13736 \end{array}$$

EXPLANATION. Multiplying a dividend by a number has the same effect upon the quotient as dividing the divisor by that number. Also, to "bring down" a figure is equivalent to multiplying the remainder by 10 and adding that number, and to "cut off" a figure from the divisor is equivalent to subtracting that number and dividing by 10. Then, evidently one may "cut off" a figure from the divisor instead of "bringing down" one from the dividend, without having much effect upon the quotient. (What is the difference?) In the work above, 16 was cut off before beginning the division, for it was at once evident that only three figures were required in the answer. The 4 of the divisor was cut off before the second division, then the 1 to its left before the third division.

EXERCISES.

Find the products, approximated to tenths, in the following :

- | | |
|---------------------------|----------------------------|
| 1. 34.6×38.42 . | 6. 26.83×3.1416 . |
| 2. 9.362×84.52 . | 7. 879.6×1.732 . |
| 3. 62.96×3.141 . | 8. 89.07×1.414 . |
| 4. 8.695×6.843 . | 9. 32.61×4.208 . |
| 5. 36.81×49.62 . | 10. 396.2×16.84 . |

Find the quotients, approximated to tenths, in the following :

- | | |
|----------------------------|-------------------------|
| 11. $96.285 \div 43.265$. | 16. $8 \div .0562$. |
| 12. $396.28 \div 3.1416$. | 17. $8 \div 1.065$. |
| 13. $84.305 \div 9.6281$. | 18. $14 \div 3.246$. |
| 14. $1.4260 \div .06345$. | 19. $256 \div 3.1416$. |
| 15. $32.465 \div .13462$. | 20. $85 \div 1.732$. |

CHAPTER II.

POWERS AND ROOTS.

116. Powers. The numbers whose product is a given number are its **factors**.

Thus, the factors of 42 are 2, 3, and 7, since $2 \times 3 \times 7 = 42$. What are the factors of 20?

If all of the factors of a number are equal, the number is called a **power** of one of the factors.

Thus, since $8 = 2 \times 2 \times 2$, 8 is a power of 2. 9 is a power of what? 125 of what?

For brevity a power is usually written in a special form. Only one of the factors is written, and a small number, called an **exponent**, is placed to the right and slightly above this factor to indicate how many times it is to be used.

Thus, 8, or $2 \times 2 \times 2$, is written 2^3 . And 81, or $3 \times 3 \times 3 \times 3$, is written 3^4 . What is the meaning of 5^2 ? Of 11^5 ?

Different powers are given special names, according to the number of factors, or, what is the same thing, the exponent.

Thus, 3^2 is read "3 to the second power," or "3 square ;"

3^3 is read "3 to the third power," or "3 cube ;"

3^4 is read "3 to the fourth power ;"

3^5 is read "3 to the fifth power ;" etc.

117. Roots. One of the equal factors of a number is called a **root** of it.

Thus, since $64 = 4 \times 4 \times 4$, 4 is a *root* of 64. What is a root of 9? Of 27? Of 125? We see, then, that that number which, when raised to a power, gives a given number is a root of the given number. Since $7^2 = 49$, 7 is a root of 49.

If the root is one of the 2 equal factors, it is the *square root*; if one of the 3 equal factors, it is the *cube root*; if one of the 4 equal factors, it is the *fourth root*; etc.

Thus, 2 is the *square root* of 4, the *cube root* of 8, the *sixth root* of 64, etc. Likewise, 5 is the *fourth root* of 625.

The root of a number is usually indicated by placing before it the sign $\sqrt{}$, called the **radical sign**.

Thus, the cube root of 343 is written $\sqrt[3]{343}$; the fourth root of 81 is written $\sqrt[4]{81}$; etc.

The number in small type written above the radical sign is called the **index** of the root, and tells what root it is, that is, determines the *order* of the root. If it is the square root, the index is omitted.

Thus, $\sqrt{64}$, $\sqrt[3]{64}$, $\sqrt[4]{64}$, are the square root, cube root, and fourth root, respectively, of 64.

118. Finding the roots of a number. It follows from the definition of a root that :

To find a required root of a number, factor it into as many equal factors as the order of the root requires, and take one of them.

Thus, to obtain the cube root of 1728, we have $1728=12 \times 12 \times 12$. Hence, $\sqrt[3]{1728}=12$.

ORAL EXERCISES.

Find the following indicated roots :

- | | | | | |
|-------------------|---------------------|----------------------|----------------------|------------------------|
| 1. $\sqrt{9}$ | 8. $\sqrt{64}$ | 15. $\sqrt[6]{64}$ | 22. $\sqrt{900}$ | 29. $\sqrt{64}$ |
| 2. $\sqrt{25}$ | 9. $\sqrt{100}$ | 16. $\sqrt[3]{1000}$ | 23. $\sqrt[3]{8000}$ | 30. $\sqrt{6.25}$ |
| 3. $\sqrt{81}$ | 10. $\sqrt[3]{125}$ | 17. $\sqrt{225}$ | 24. $\sqrt{2025}$ | 31. $\sqrt[3]{.008}$ |
| 4. $\sqrt[3]{27}$ | 11. $\sqrt[3]{216}$ | 18. $\sqrt[3]{512}$ | 25. $\sqrt[6]{729}$ | 32. $\sqrt[3]{1.728}$ |
| 5. $\sqrt[3]{64}$ | 12. $\sqrt[4]{81}$ | 19. $\sqrt{1225}$ | 26. $\sqrt[8]{256}$ | 33. $\sqrt[3]{.729}$ |
| 6. $\sqrt{36}$ | 13. $\sqrt[5]{32}$ | 20. $\sqrt[4]{1296}$ | 27. $\sqrt{.01}$ | 34. $\sqrt[4]{.0001}$ |
| 7. $\sqrt{49}$ | 14. $\sqrt[3]{343}$ | 21. $\sqrt[5]{1024}$ | 28. $\sqrt{1.44}$ | 35. $\sqrt[5]{.00032}$ |

119. Roots of common fractions. Since $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{16}{81}$, $\sqrt[4]{\frac{16}{81}} = \frac{2}{3}$. Since $(\frac{4}{5})^3 = \frac{64}{125}$, $\sqrt[3]{\frac{64}{125}} = \frac{4}{5}$. And in general, a root of a fraction is obtained by finding the roots of its numerator and denominator for the numerator and denominator, respectively, of the root. This may be expressed thus:

$$\sqrt[n]{\frac{n}{d}} = \frac{\sqrt[n]{n}}{\sqrt[n]{d}}.$$

ORAL EXERCISES.

Find the roots as indicated:

- | | | | |
|----------------------------|-------------------------------|----------------------------------|---------------------------------|
| 1. $\sqrt{\frac{4}{9}}$ | 6. $\sqrt{\frac{4}{81}}$ | 11. $\sqrt[3]{\frac{125}{216}}$ | 16. $\sqrt[5]{\frac{1}{32}}$ |
| 2. $\sqrt{\frac{1}{16}}$ | 7. $\sqrt{\frac{16}{625}}$ | 12. $\sqrt[3]{\frac{1}{343}}$ | 17. $\sqrt[5]{\frac{32}{243}}$ |
| 3. $\sqrt{\frac{9}{16}}$ | 8. $\sqrt[3]{\frac{1}{27}}$ | 13. $\sqrt[4]{\frac{16}{81}}$ | 18. $\sqrt[6]{\frac{1}{64}}$ |
| 4. $\sqrt{\frac{25}{49}}$ | 9. $\sqrt[3]{\frac{8}{125}}$ | 14. $\sqrt[4]{\frac{1}{82}}$ | 19. $\sqrt[7]{\frac{128}{243}}$ |
| 5. $\sqrt{\frac{64}{100}}$ | 10. $\sqrt[3]{\frac{27}{64}}$ | 15. $\sqrt[4]{\frac{81}{10000}}$ | 20. $\sqrt[3]{\frac{3}{8}}$ |

120. Second method of finding roots. The method of finding roots by factoring is often impracticable. A second method is here given for finding square and cube roots. There are similar methods for fourth roots, fifth roots, etc., but these are so long and tedious that they are not used to-day in practical life. Such roots are easily obtained by logarithms, as shown in the next chapter. Even square and cube roots are much more easily found by logarithms.

121. Square root. The squares of the numbers 1, 2, 3, ..., 9, 10, are 1, 4, 9, ..., 81, 100, respectively. Hence, the square root of an integer of *one* or *two* figures is a number of *one* figure.

The squares of 10, 11, ..., 99, 100, are 100, 121, ..., 9801, 10000, respectively. Hence, the square root of an integer of *three* or *four* figures is a number of *two* figures.

Likewise, the square root of an integer of *five* or *six* figures is a number of *three* figures, and so on.

Therefore, if the figures of an integer are marked off from right to left in groups of two, the number of figures in the square root will be equal to the number of groups, any one figure remaining on the left being counted as a group.

Thus, marked off into groups, 26244 becomes 2'62'44'. Hence, $\sqrt{26244}$ will contain three figures.

If the square root of a number contains two figures, we may denote the *tens* of the root by a and the *ones* by b . Hence, the root will be represented by $a + b$. Then it is shown in *algebra* that $(a + b)^2 = a^2 + 2ab + b^2$. Hence the given number will be represented by $a^2 + 2ab + b^2$.

E. g., $35 = 30 + 5$. Here, $a = 30$ and $b = 5$. Then, $35^2 = 30^2 + 2 \times 30 \times 5 + 5^2 = 900 + 300 + 25 = 1225$.

The use of the formula $\sqrt{a^2 + 2ab + b^2} = a + b$ in finding square roots is best shown by examples. For a fuller treatment of the subject, the student is referred to the Stone-Millis *Essentials of Algebra*, Appendix.

EXAMPLE 1. Find the square root of 2116.

Pointing off, we have 21'16'. Hence, the root contains two figures. Since 40^2 is less than 2116, and 50^2 is greater than 2116, the root must lie between 40 and 50; *i. e.*, the *tens' figure of the root* is 4, the *square root of the largest square in the left-hand group of the given number*. Hence, in the above formula, a denotes 4 *tens*, or 40. The work may be performed as follows:

	$a + b$
	21'16' (40 + 6 = 46, root)
$a^2 =$	16 00
$2a = 80$	5 16 = $2ab + b^2$
$2a + b = 86$	
$2ab + b^2 =$	5 16

EXPLANATION. Since $a = 40$, $a^2 = 1600$. Subtracting 1600, the remainder is 516, represented by $2ab + b^2$. Now since *tens* are much larger relatively than *ones*, the part $2ab$ of the remainder is much larger than b^2 . Hence, b , the *ones' figure*, may be found approximately by dividing the remainder by $2a$, or *twice the root found*. $2a$ is called the **trial divisor**, and becomes 80. Dividing 516 by 80 gives approximately 6. Hence b is probably 6

Adding this to the trial divisor gives the **complete divisor** $2a+b$, which is 86. Multiplying 86 by 6 gives 516, represented by $2ab+b^2$. Subtracting this from 516, the first remainder, leaves zero. Hence, $40+6$, or 46, is the square root.

By similar reasoning, it will be seen that when the root is a number of three figures, the first two may be found as above; then the number which they form may be used as one, and the third figure found by a repetition of the process used to obtain the second. Thus the process may be extended to any number of figures.

NOTE.—In practice, after learning the process, the student may omit writing the formulae in the work, and use the abbreviated form shown in the next example.

EXAMPLE 2. Find the square root of 138384.

Pointing off, we have 13'83'84'. Hence, there are three figures in the root.

$$\begin{array}{r}
 13'83'84' \overline{)300 + 70 + 2 = 372, \text{ root}} \\
 \underline{9 \ 00 \ 00} \\
 600 \overline{)4 \ 83 \ 84} \\
 \underline{670} \quad \overline{)4 \ 69 \ 00} \\
 740 \quad \overline{)14 \ 84} \\
 \underline{742} \quad \overline{)14 \ 84}
 \end{array}$$

EXPLANATION. The largest square in 13 is 9. Hence the *hundreds* part of the root is 300. Subtracting 90000 leaves 48384. The *first trial divisor* is 2×300 , or 600. Dividing 48384 by 600 gives approximately 70. Adding 70 to 600 gives 670, the *complete divisor*. Multiplying this by 70 gives 46900; and subtracting 46900 from 48384 leaves 1484 for second remainder. Now adding 300 and 70, and multiplying by 2, gives 740 for the *second trial divisor*. Dividing 1484 by 740 gives approximately 2, the third figure in the root. Hence, the *second complete divisor* is 742. Multiplying this by 2, we get 1484, and subtracting this from the second remainder leaves 0. Hence, the required root is 372.

In extracting square roots of decimals, use is made of the principle that

When the square root of a number has decimal places, the number itself has twice as many.

Thus, $.4^2 = .16$, $.25^2 = .0625$, $.125^2 = .015625$.

Therefore, to mark off a number which contains decimal places, begin at the decimal point and mark both to the left and to the right, putting two figures in each group.

If necessary, a cipher may be annexed to the decimal to make a full period.

E. g., 723.618 will become 7'23'.61'80'.

EXAMPLE 3. Find the square root of 50.9796.

$$\begin{array}{r}
 50'.97'96')7.00+.10+.04=7.14, \text{ root} \\
 \underline{49.00\ 00} \\
 14.00 \mid 1.97\ 96 \\
 \underline{14.10 \mid 1.41\ 00} \\
 14.20 \mid .56\ 96 \\
 \underline{14.24 \mid .56\ 96}
 \end{array}$$

An approximation to the value of the square root of a number which is not a perfect square may be obtained to any degree of accuracy desired. Most problems in square root met in practical life are those in which only approximate values of the roots can be found.

EXAMPLE 4. Find to three decimal places the square root of 2.

Since we want three decimal places, it is convenient to annex 6 ciphers. The work is as follows :

$$\begin{array}{r}
 2'.00'00'00')1.+.4+.01+.004=1.414 \\
 \underline{1.00\ 00\ 00} \\
 2.000 \mid 1.00\ 00\ 00 \\
 \underline{2.400 \mid .96\ 00\ 00} \\
 2.800 \mid .04\ 00\ 00 \\
 \underline{2.810 \mid .02\ 81\ 00} \\
 2.820 \mid .01\ 19\ 00 \\
 \underline{2.824 \mid .01\ 12\ 96} \\
 .00\ 06\ 04
 \end{array}$$

In practice, the student will soon learn to omit the ciphers in the calculation.

If both terms of a *common fraction* are not perfect squares, so that its square root may not be obtained as in § 119, the

fraction may first be reduced to the decimal form, then the square root of this decimal extracted.

* Thus, $\sqrt{\frac{1}{2}} = \sqrt{.6} = .774 +$.

Or the fraction may be reduced to an equal fraction whose denominator is a square number.

Thus, $\sqrt{\frac{1}{2}} = \sqrt{\frac{2}{4}} = \frac{1}{2} \sqrt{2} = \frac{1}{2}$ of 1.4142 + = .707 +.

EXERCISES.

Extract the square root of:

- | | | | | |
|----------|-----------|-------------|--------------|----------------|
| 1. 784 | 5. 5329. | 9. 173056. | 13. 14641. | 17. 2611.21. |
| 2. 1849. | 6. 6724. | 10. 492804. | 14. 12.8881. | 18. .026244. |
| 3. 3969. | 7. 54756. | 11. 34225. | 15. 125.44. | 19. .055696. |
| 4. 1521. | 8. 41209 | 12. 12.25. | 16. 4.6225. | 20. .00174724. |

Find to three decimal places the square root of:

- | | | | | | |
|---------|----------|---------------------|----------------------|----------------------|-----------------------|
| 21. 3. | 25. 2. | 29. 35. | 33. $\frac{1}{6}$. | 37. $\frac{5}{16}$. | 41. $\frac{1}{128}$. |
| 22. 5. | 26. .31. | 30. 1.04. | 34. $\frac{5}{7}$. | 38. $\frac{6}{19}$. | 42. $\frac{1}{25}$. |
| 23. 12. | 27. 6.2. | 31. $\frac{2}{3}$. | 35. $\frac{3}{11}$. | 39. $\frac{1}{25}$. | 43. $\frac{4}{27}$. |
| 24. 7. | 28. .06. | 32. $\frac{7}{8}$. | 36. $\frac{3}{4}$. | 40. $\frac{9}{17}$. | 44. $\frac{4}{18}$. |

122. Cube root. This method of finding cube root is so little used in practical life that it is not worth while to study it if one understands the use of logarithms. It is placed here for those who wish to take it and who will omit the next chapter. The method is as follows :

The cubes of the numbers 1, 2, 3,..... 9, 10, are 1, 8, 27,..... 729, 1000, respectively. Hence, the cube root of an integer of *one, two* or *three* figures is a number of *one* figure.

The cubes of the numbers 10, 11,.....99, 100, are 1000, 1831,.....970290, 1000000, respectively. Hence, the cube root of an integer of *four, five* or *six* figures is a number of *two* figures.

Likewise, the cube root of an integer of *seven, eight* or *nine* figures is a number of *three* figures. And so on.

Therefore, if the figures of an integer are marked off from right to left in groups of three, the number of figures in the cube root will be equal to the number of groups, one or two figures remaining on the left being counted as a group.

Thus, marked off into groups, 2515456 becomes 2'515'456'. Since there are three groups, the cube root of 2515456 will contain three figures, i. e., ones, tens and hundreds.

If the cube root of a number be a number of two figures, we may denote the tens of the root by a and the ones by b , and hence, represent the root by $a+b$. It is shown in algebra that $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. Hence, the given number is represented by $a^3 + 3a^2b + 3ab^2 + b^3$.

Therefore, the cube root of the number may be obtained by use of the identity

$$\sqrt[3]{a^3 + 3a^2b + 3ab^2 + b^3} = a + b.$$

The process is best shown by an example.

EXAMPLE. Find the cube root of 39304.

Pointing off, we have 39'304'. Hence, the root will be a number of two figures. Since 30^3 is less than 39304, and 40^3 is greater than 39304, the root lies between 30 and 40; i. e., the tens' figure of the root is 3, the cube root of the greatest cube in the left-hand group of the given number. Therefore, in the above identity a denotes 3 tens, or 30.

The work is then performed as follows:

$a^3 =$	39'304'	$a+b$
	27 000	$30+4=34$, root
$3a^2 = 2700$	12304	
$3a^2 + 3ab + b^2 = 3076$		
$3a^2b + 3ab^2 + b^3 =$	12304	

Since $a=30$, $a^3=27000$. Subtracting 27000 leaves 12304. The trial divisor, $3a^2$, becomes 2700. Dividing 12304 by 2700 gives approximately 4. Hence, b is probably 4. The complete divisor, $3a^2 + 3ab + b^2$, becomes 3076. Multiplying 3076 by 4 gives 12304. Subtracting this from the first remainder leaves 0. Hence, 30+4, or 34, is the required root.

When the root is a number of three figures the first two may be found as above; then their sum may be used as one number, and the third obtained by a repetition of the process used to obtain the second. And so on.

The cube root of a decimal fraction. For reasons similar to those given in § 121,

To mark off a number which contains a decimal, begin at the decimal point and mark to the left and to the right, putting three figures in each group.

If necessary, ciphers may be annexed to the right of the decimal point.

An approximate value of the cube root of a number which is not a perfect cube may be obtained to any required degree of accuracy.

NOTE.— a and a' denote the part of the root found. b and b' denote each new term.

EXAMPLE. Find the cube root of 1860.867.

	1'860'.867'	10. + 2. + .3 = 12.3, root
$a^3 =$	1 000.	
$3a^2 = 300.$	860.867	
$8a^2 + 3ab + b^2 = 364.$		
$3a^2b + 3ab^2 + b^3 =$	728.	
$3a^2 = 432.$	132.867	
$8a^2b + 3a'b^2 + b'^2 = 442.89$		
$8a^2b' + 3a'b'^2 + b'^3 =$	132.867	

EXERCISES.

Find the cube root of :

1. 108828. 2. 2744. 3. 15625. 4. 148877. 5. 571787.
6. 340068392. 7. 523606616. 8. 59.319. 9. 328.509. 10. 41.063625

Find to two decimal places the cube root of :

11. 2. 12. 10. 13. 106. 14. 3.7. 15. .15.

CHAPTER III.

LOGARITHMS.

123. Logarithms. In practical computation, extensive use is made of numbers called **logarithms**. By their use multiplication may be performed by a single addition, division by a single subtraction, the power of a number obtained by a single multiplication, and the root of a number by a single division.

NOTE.—Since the subject of logarithms is merely to give a method of computation that will save much work, it may be omitted from short courses or by those who prefer to compute by the methods already given.

The *logarithm of a number* is the exponent which indicates the power to which another number must be raised to produce that number. The number which is raised to the power is called the **base** of the logarithm.

Thus, since $2^3 = 8$, 3 is a *logarithm* of 8, and 2 the *base*. This is expressed by $\log_2 8 = 3$. Since $10^2 = 100$, 2 is the *logarithm* of 100 to the *base* 10. This relation is expressed by $\log_{10} 100 = 2$. Likewise, $\log_5 625$ is read “logarithm of 625 to the base 5,” and equals 4, because $5^4 = 625$.

In general, if $a^n = b$, then n is the logarithm of b to the base a ; and is expressed by $\log_a b = n$.

EXERCISES.

Express the following relations in terms of logarithms :

- | | | |
|------------------|----------------------|---------------------|
| 1. $2^4 = 16$. | 5. $4^3 = 64$. | 9. $5^6 = 15625$. |
| 2. $2^7 = 128$. | 6. $7^4 = 2401$. | 10. $6^3 = 216$. |
| 3. $3^2 = 9$. | 7. $10^5 = 100000$. | 11. $20^2 = 400$. |
| 4. $5^3 = 125$. | 8. $10^1 = 10$. | 12. $12^3 = 1728$. |

Express the following relations by means of exponents:

- | | | |
|-----------------------|----------------------------|---------------------------|
| 13. $\log_2 32 = 5$. | 17. $\log_3 27 = 3$. | 21. $\log_{10} 10 = 1$. |
| 14. $\log_3 9 = 2$. | 18. $\log_4 64 = 3$. | 22. $\log_2 216 = 3$. |
| 15. $\log_2 16 = 4$. | 19. $\log_5 25 = 2$. | 23. $\log_{11} 144 = 2$. |
| 16. $\log_4 16 = 2$. | 20. $\log_{10} 1000 = 3$. | 24. $\log_7 343 = 3$. |

Find the values of:

- | | | |
|--------------------|-------------------------|---------------------------|
| 25. $\log_2 16$. | 31. $\log_5 36$. | 37. $\log_{10} 10000$. |
| 26. $\log_5 512$. | 32. $\log_5 512$. | 38. $\log_{11} 1000000$. |
| 27. $\log_3 243$. | 33. $\log_5 625$. | 39. $\log_{10} 10^{13}$. |
| 28. $\log_2 81$. | 34. $\log_{15} 225$. | 40. $\log_{11} 1728$. |
| 29. $\log_3 125$. | 35. $\log_{10} 27000$. | 41. $\log_{11} 121$. |
| 30. $\log_4 64$. | 36. $\log_{10} 1000$. | 42. $\log_{14} 14^3$. |

124. Fundamental principles in exponents. Certain principles in exponents which are involved in the use of logarithms are here given.

$3^2 \times 3^4 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$. $a^3 \times a^2 = a \times a \times a \times a \times a = a^5$.
In general,

I. *The exponent of the product of two powers of the same number is the sum of the exponents of the powers.*

Since $2^3 \times 2^2 = 2^5$, $2^5 + 2^2 = 2^7$. Since $a^4 \times a^5 = a^9$, $a^9 + a^5 = a^4$. In general,

II. *The exponent of the quotient of two powers of the same number is the exponent of the dividend minus the exponent of the divisor.*

$(4^5)^3 = 4^5 \times 4^5 \times 4^5 = 4^{15}$. $(a^2)^4 = a^2 \times a^2 \times a^2 \times a^2 = a^8$. In general,

III. *The exponent of a power of a power of a number is the product of the exponents.*

Since $(2^3)^4 = 2^{12}$, $\sqrt[4]{2^{12}} = 2^3$. Since $(a^5)^2 = a^{10}$, $\sqrt[5]{a^{10}} = a^2$. In general,

IV. *The exponent of a root of a power of a number is the quotient of the exponent of the number divided by the index of the root.*

EXERCISES.

Express with one exponent :

- | | | | |
|-----------------------------------|--------------------------------|----------------------------|--------------------|
| 1. $2^6 \times 2^9$. | 9. $a \times a^5 \times a^7$. | 17. $6^{30} \div 6^{15}$. | 25. $(7^3)^3$. |
| 2. $4^7 \times 4^{12}$. | 10. $a^{20} \times a^{15}$. | 18. $a^9 \div a^4$. | 26. $(4^{20})^6$. |
| 3. $6^2 \times 6^3$. | 11. $4^6 \div 4^2$. | 19. $a^5 \div a$. | 27. $(a^9)^3$. |
| 4. $2^3 \times 2^4 \times 2^5$. | 12. $3^{12} \div 3^7$. | 20. $a^8 \div a^7$. | 28. $(a^5)^7$. |
| 5. $5 \times 5^3 \times 5^5$. | 13. $5^{18} \div 5^{10}$. | 21. $a^{21} \div a^{18}$. | 29. $(a^2)^{10}$. |
| 6. $8^4 \times 8^{10} \times 8$. | 14. $2^{14} \div 2^6$. | 22. $(6^2)^6$. | 30. $(a^{13})^7$. |
| 7. $a^6 \times a^4 \times a^2$. | 15. $7^{13} \div 7^5$. | 23. $(2^5)^2$. | 31. $(a^4)^4$. |
| 8. $a^2 \times a^9 \times a^3$. | 16. $10^{12} \div 10^{10}$. | 24. $(5^2)^4$. | 32. $(a^6)^7$. |

Express merely with an exponent :

- | | | | |
|--------------------------|--------------------------|---------------------------|---------------------------|
| 33. $\sqrt[3]{2^{12}}$. | 37. $\sqrt[3]{9^{14}}$. | 41. $\sqrt{8^{10}}$. | 45. $\sqrt[4]{a^{80}}$. |
| 34. $\sqrt[4]{5^8}$. | 38. $\sqrt[5]{2^{25}}$. | 42. $\sqrt[3]{10^{15}}$. | 46. $\sqrt[4]{a^{16}}$. |
| 35. $\sqrt[6]{3^{18}}$. | 39. $\sqrt[4]{8^{24}}$. | 43. $\sqrt[9]{2^{72}}$. | 47. $\sqrt[5]{a^{35}}$. |
| 36. $\sqrt[4]{4^6}$. | 40. $\sqrt[8]{4^{24}}$. | 44. $\sqrt[4]{a^{28}}$. | 48. $\sqrt[10]{a^{90}}$. |

125. Zero and negative exponents.

By Principle II, § 124, $2^4 + 2^7 = 2^4 - 7$, which is written 2^{-7} . But $2^4 + 2^7 = \frac{1}{2^3}$. Hence, 2^{-7} means $\frac{1}{2^3}$. Likewise $a^2 + a^3 = a^{-6}$ which means $\frac{1}{a^6}$.

In general, a^{-n} means $\frac{1}{a^n}$.

These exponents with minus signs before them are called **negative exponents**.

Again, by the same principle, $3^4 + 3^4 = 3^0$. But $3^4 + 3^4 = 1$. Hence $3^0 = 1$. Likewise $10^6 + 10^6 = 10^0 = 1$.

In general, a^0 means 1.

Since $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$, $\log_{\frac{1}{16}} = -2$. Since $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$, $\log_{\frac{1}{81}} = -4$.

In general, if the base is a whole number, the logarithm of a number less than 1 is negative.

Since $4^0 = 1$, $\log_4 1 = 0$. Since $a^0 = 1$, $\log_a 1 = 0$.

The logarithm of 1 to any base is 0.

126. Fundamental principles in logarithms. The following useful principles in logarithms follow directly from the principles in § 124.

I. *The logarithm of the product of two numbers equals the sum of their logarithms.*

For, let $a^x = m$, and $a^y = n$.

Then $m \times n = a^x \times a^y = a^{x+y}$.

Hence, $\log_a m \times n = x + y = \log_a m + \log_a n$.

II. *The logarithm of a quotient equals the logarithm of the dividend minus the logarithm of the divisor.*

For, let $a^x = m$, and $a^y = n$.

Then, $\frac{m}{n} = a^x \div a^y = a^{x-y}$.

Hence, $\log_a \frac{m}{n} = x - y = \log_a m - \log_a n$.

III. *The logarithm of a power of a number equals the logarithm of the number, multiplied by the exponent of the power*

For, let $a^x = n$.

Then, $n^p = (a^x)^p = a^{px}$.

Hence, $\log_a n^p = px = p \log_a n$.

IV. *The logarithm of a root of a number equals the logarithm of the number, divided by the index of the root.*

For let $a^x = n$.

Then, $\sqrt[r]{n} = \sqrt[r]{a^x} = a^{\frac{x}{r}}$.

Hence, $\log_a \sqrt[r]{n} = \frac{x}{r} = \frac{\log_a n}{r}$, or $\frac{1}{r} \log_a n$.

EXAMPLE 1. $\log_{10} 30 = \log_{10} 2 \times 3 \times 5 = \log_{10} 2 + \log_{10} 3 + \log_{10} 5$.

EXAMPLE 2. $\log_{10} \frac{1}{2} = \log_{10} 5 - \log_{10} 7$.

EXAMPLE 3. $\log_{10} 19^8 = 8 \log_{10} 19$.

EXAMPLE 4. $\log_{10} \sqrt[3]{5} = \frac{1}{3} \log_{10} 5$.

EXERCISES.

1. Show that $\log_{10} 15 - \log_{10} 6 + \log_{10} \frac{2}{3} = 0$.

2. Show that $\log_{10} \sqrt{20} - \frac{1}{2} \log_{10} 5 = \log_{10} 2$.

3. Show that $2 \log_{10} 10 - \log_{10} 4 = \log_{10} 25$.

If $\log_{10} 2 = .3010$, $\log_{10} 3 = .4771$, $\log_{10} 5 = .6990$, $\log_{10} 7 = .8451$, find to the base 10 the logarithms of the following:

- | | | | | |
|---|---|---|----------------------|-----------------------|
| 4. 18. | 7. 50. | 10. $1\frac{2}{3}$. | 13. 1000. | 16. $\sqrt{30}$. |
| 5. 15. | 8. 24. | 11. 70. | 14. $6\frac{2}{3}$. | 17. $\sqrt[3]{180}$. |
| 6. 20. | 9. $2\frac{1}{3}$. | 12. 210. | 15. $\sqrt{18}$. | 18. $\sqrt[4]{270}$. |
| 19. $\frac{\sqrt{98}}{\sqrt[3]{144}}$. | 20. $\frac{\sqrt{2} \times \sqrt[3]{3}}{\sqrt[3]{5} \times \sqrt{7}}$. | 21. $\frac{(5\frac{3}{8})^3}{(5\frac{5}{8})^3}$. | | |

127. Common logarithms. The logarithm of a number to the base 10 is called a **common logarithm**.* In all practical computations, common logarithms are used, because the base 10 is also the base of our decimal system of numbers. The remainder of this chapter will be confined to common logarithms.

It is customary, in using common logarithms, to omit writing the base, the base 10 being understood.

Thus, $\log_{10} 12.6$ is written simply $\log 12.6$.

* The system of common logarithms was introduced in the seventeenth century by Henry Briggs. Accordingly, logarithms to the base 10 are also called **Briggs' logarithms**. Another important system is the **Napierian system**, named after Napier, a contemporary of Briggs. The base of the Napierian system is the sum of the infinite series $1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots$. The sum of this series is approximately 2.7182818, and is represented by the letter e . Napier himself, however, did not use the base e in his system.

While the common system is used in practical calculations, the Napierian system is used in theoretical investigations.

128. Characteristic and mantissa. From the definition of a logarithm :

since $10^0=1$, $\log 1=0$;

since $10^1=10$, $\log 10=1$;

since $10^{-1}=.1$, $\log .1=-1$;

since $10^2=100$, $\log 100=2$;

since $10^{-2}=.01$, $\log .01=-2$;

since $10^3=1000$, $\log 1000=3$;

since $10^{-3}=.001$, $\log .001=-3$; etc.

Moreover, since 42 is between 10 and 100, $\log 42$ is between 1 and 2; *i. e.*, $\log 42=1$ and a decimal. Likewise, since 689 is between 100 and 1000, $\log 689$ is between 2 and 3; *i. e.*, $\log 689=2$ and a decimal. In general, the logarithm of any number that is not a power of 10 consists of *an integral and a decimal part*.

Thus, $\log 825=2.9165$. It should be remembered that the value of the decimal part can be obtained only approximately, but may be computed to any degree of accuracy desired.

The integral part of a logarithm is called its **characteristic**, and the decimal part is called its **mantissa**.

129. How to find the characteristic. The characteristic may be found by inspection.

Thus, since 428.3 is between 100 and 1000, $\log 428.3$ is between 2 and 3; *i. e.*, $\log 428.3=2+$ some mantissa.

Since 4.25 is between 1 and 10, $\log 4.25$ is between 0 and 1; *i. e.*, $\log 4.25=0+$ some mantissa.

Since .00328 is between .01 and .001, $\log .00328$ is between -2 and -3; *i. e.*, $\log .00328=-3+$ some mantissa.

Since .45 is between 1 and .1, $\log .45$ is between 0 and -1; *i. e.*, $\log .45=-1+$ some mantissa.

It is clear that the characteristic may be found by the following rules:

I. If a number is greater than 1, the characteristic of its logarithm is less by 1 than the number of figures to the left of the decimal point.

II. *If a number is less than 1, the characteristic of its logarithm is negative, and is greater by 1 than the number of zeros immediately to the right of the decimal point.*

If the characteristic is negative, its value is usually subtracted from a multiple of 10, the mantissa annexed to the result, and the multiple of 10 indicated to be subtracted from the final result.

Thus, in $\log .00062$, the characteristic is -4 and the mantissa .7924. This is written $6.7924-10$. Similarly, $\log .00000000000396=8.5977-20$.

130. *Logarithms of numbers which differ only in the positions of the decimal point have the same mantissa.*

This follows from the fact that changing the position of the decimal point is equivalent to multiplying or dividing the number by some power of 10, by which an integer is added to or subtracted from its logarithm, leaving the mantissa unchanged.

Thus, $4.237 \times 10 = 42.37$; $4.237 \times 10^2 = 423.7$; $4.237 \div 10^2 = .04237$.

And $\log 8230 = 3.9154$; $\log 82.3 = 1.9154$; $\log .0823 = 8.9154 - 10$.

131. Tables of mantissas. The mantissas of the logarithms of consecutive integers have been computed and tabulated. At the end of this chapter is a table which contains the mantissas of the logarithms of all integers from 1 to 1000, computed to four decimal places.

In this table, the first two figures of each number are given in the column headed *N*, and the third figure in the horizontal line at the top of the table. The corresponding mantissas, with decimal points omitted, are given in the columns headed 0, 1, 2, 3, etc.

To find the logarithm of a number, find its characteristic by §129, and look in the table for the mantissa. In looking for the mantissa of a number containing less than three figures, annex ciphers until it has three figures.

Thus, to find the mantissa of $\log 86$, look for the mantissa of $\log 860$. To find the mantissa of $\log 4$, look for the mantissa of $\log 400$.

132. Use of the table: To find the logarithm of a given number.

I. *When the given number contains not more than three figures*, the mantissa of its logarithm is obtained directly from the table.

EXAMPLE 1. Find $\log 57.4$.

By § 129, the characteristic is 1. The mantissa is the same as the mantissa of $\log 574$.

Look for 57 in the column headed N. Looking along the horizontal line of numbers opposite 57, to the column headed 4, we find .7589. Hence, $\log 57.4 = 1.7589$.

EXAMPLE 2. Find $\log .089$.

The characteristic is -2 . The mantissa is the same as that of $\log 890$. This is opposite 89, in the column headed 0, and is .9494. Hence, $\log .089 = 8.9494 - 10$.

II. *When the given number contains more than three figures*, use the principle that when the difference of two numbers is small compared with either of them, the difference of the numbers is approximately proportional to the difference of their logarithms.

EXAMPLE 3. Find $\log 6.3172$.

The characteristic is 0.

The required mantissa is the same as that of $\log 631.72$.

Now 631.72 is greater than 631 by .72.

The mantissa of $\log 631 = .8000$.

The mantissa of $\log 632 = .8007$.

Subtracting, $.8007 - .8000 = .0007$.

Hence, an increase of 1 in 631 causes an increase of .0007 in the corresponding mantissa. Therefore, an increase of .72 will cause an increase of $.72 \times .0007$, or approximately .0005, in the mantissa.

Adding .0005 to the mantissa of $\log 631$ gives .8005.

Hence, $\log 6.3172 = 0.8005$.

EXAMPLE 4. Find $\log .006418$.

The characteristic is -3 .

The required mantissa is that of $\log 641.8$.

Mantissa of $\log 641 = .8069$.

Mantissa of $\log 642 = .8075$.

$$.8075 - .8069 = .0006.$$

$$.8 \times .0006 = .0005.$$

$$.8069 + .0005 = .8074.$$

Hence, $\log .006418 = 7.8074 - 10$.

EXERCISES.

Find the logarithms of:

- | | | | | |
|----------|--------------|----------|------------|--------------|
| 1. 327. | 9. 62.5. | 17. 900. | 25. 1. | 33. 3.1416. |
| 2. 764. | 10. .842. | 18. 260. | 26. 68. | 34. 65321. |
| 3. 925. | 11. 19.9. | 19. 470. | 27. 4500. | 35. 27.342. |
| 4. 183. | 12. 8.56. | 20. 600. | 28. 32600. | 36. 5.3219. |
| 5. 42.7. | 13. .613. | 21. 2. | 29. 4286. | 37. .6285. |
| 6. 3.94. | 14. .0869. | 22. 37. | 30. 9135. | 38. .007932. |
| 7. 286. | 15. .00321. | 23. 49. | 31. 62.84. | 39. .063805. |
| 8. 9.28. | 16. .000914. | 24. 7. | 32. 526.7. | 40. 4268.4. |

133. To find a number whose logarithm is given.

I. *When the given mantissa is found in the table*, the sequence of figures of the required number may be obtained by reversing the process of Case I, § 132. The position of the decimal point is determined by reversing the rules in § 129.

EXAMPLE 1. Find the number whose logarithm is $8.7649 - 10$.

In the table, the mantissa .7649 is opposite 58 and in the column headed 2. Hence, $.7649 =$ mantissa of $\log 582$.

Since the characteristic is -2 , the number must be less than 1, and must contain one zero immediately to the right of the decimal point.

Hence, $8.7649 - 10 = \log .0582$.

II. *When the given mantissa is not found in the table*, the

required number is obtained by reversing the process of Case II, § 132.

EXAMPLE 2. Find the number whose logarithm is 1.5784.

Mantissa .5784 is not in the table, but those mantissas of the table between which it lies are .5729 and .5740.

.5729 = mantissa of log 874, and .5740 = mantissa of log 875.

Subtracting, $.5740 - .5729 = .0011$,

and $.5784 - .5729 = .0005$.

Hence, an increase of .0011 in the mantissa causes an increase of 1 in the corresponding number. Therefore, an increase of .0005 in the mantissa will cause an increase of $.0005 \div .0011$, or .45, in the number.

Adding .45 to 874 gives 874.45.

Therefore, .5784 = mantissa of log 874.45.

Since the characteristic is 1, we have $1.5784 = \log 87.445$.

EXERCISES.

Find the numbers whose logarithms are:

- | | | | |
|---------------|----------------|----------------|----------------|
| 1. 1.3248. | 8. 3.4409. | 15. 5.7412—10. | 22. 8.4704—10. |
| 2. 3.5416. | 9. 0.6875. | 16. 0.6085. | 23. 4.0150. |
| 3. 4.9227. | 10. 2.8609. | 17. 2.4975. | 24. 0.4735. |
| 4. 9.7686—10. | 11. 9.7443—10. | 18. 1.1852. | 25. 1.6500. |
| 5. 7.8142—10. | 12. 8.0086—10. | 19. 3.7391. | 26. 8.8187—10. |
| 6. 2.5490. | 13. 1.9773. | 20. 9.7585—10. | 27. 2.9989. |
| 7. 6.6794. | 14. 7.8363—10. | 21. 1.9260. | 28. 9.4545—10. |

134. Computation by logarithms. By use of the principles in § 126, and the processes of § 132 and § 133, the student may make any of the complicated computations that he will ever have to make in practical life. Logarithms should be used in all of the problems in the following chapters that involve long computations in the applications of arithmetic to mensuration and science.

EXAMPLE 1. Raise 1.78 to the fifth power.

We first find $\log 1.78^5$.

$$\begin{aligned}\text{By § 126, III, } \log 1.78^5 &= 5 \times \log 1.78 \\ &= 5 \times .2504 \\ &= 1.2520\end{aligned}$$

Now, finding the number of which 1.2520 is the logarithm,

$$1.2520 = \log 17.86.$$

Hence, $1.78^5 = 17.86$, approximately.

EXAMPLE 2. Find the cube root of .0427.

First, find $\log \sqrt[3]{.0427}$.

$$\begin{aligned}\text{By § 126, IV, } \log \sqrt[3]{.0427} &= \frac{1}{3} \text{ of } \log .0427 \\ &= \frac{1}{3} \text{ of } (8.6304 - 10) \\ &= \frac{1}{3} \text{ of } (28.6304 - 30) \\ &= 9.5435 - 10.\end{aligned}$$

But, $9.5435 - 10 = \log .3495$.

Hence, $\sqrt[3]{.0427} = .3495$, approximately.

EXAMPLE 3. Find the value of $\sqrt{\frac{2.47 \times 73.84}{52.4}}$.

$$\log \sqrt{\frac{2.47 \times 73.84}{52.4}} = \frac{\log 2.47 + \log 73.84 - \log 52.4}{2}$$

$$\log 2.47 = .3927$$

$$\log 73.84 = 1.8683$$

$$\underline{2.2610}$$

$$\log 52.4 = 1.7198$$

$$\underline{2) .5417}$$

$$.2709 = \log 1.866.$$

Hence, $\sqrt{\frac{2.47 \times 73.84}{52.4}} = 1.866$ approximately.

EXERCISES.

Find by logarithms the value of:

1. $32.7 \times .468$.

5. $1.414 \times .0321$.

9. $\sqrt[3]{34.95}$.

2. 1.805×360 .

6. 3.89^4 .

10. $\sqrt[5]{9.034}$.

3. $25.3 \times 6.42 \times .063$.

7. $.63^6$.

11. $\sqrt[4]{.00328}$.

4. 1728×3.1416 .

8. 52.84^5 .

12. $\sqrt[3]{6.23^2}$.

13. $3.142 \div 32.6$. 17. $\frac{627 \times 892}{1256}$. 20. $\sqrt[3]{256.8 \div 1728}$.
 14. $181.2 \div 13.92$. 18. $\frac{1728}{3.52 \times 62.3}$. 21. $86.3^2 \div 1062$.
 15. $235 \times 4.27 \div 12.6$. 22. $172.8^2 \div 14.9^3$.
 16. $100 \div 3.1416$. 23. $.6814^{10}$.
 19. $\sqrt[4]{4.567 \times 32.7}$. 24. $\sqrt[5]{39.2^2 \times 4.6^3}$.

25. If the interest on P dollars is compounded annually, at the rate of r per cent for n years, the amount is given by the formula, $Amount = (1 + \frac{r}{100})^n \times P$.

Find the amount of \$425 at the end of 20 years, compounded annually at 5%.

26. Find the amount of \$100 at the end of 100 years, compounded annually at 4%.

27. If D inches is the deflection in the middle of a beam, supported at the ends and loaded in the middle, then $D = \frac{W \times L^3}{48 \times E \times I}$, where W = load in lb., L = length of the beam in inches, and E and I are constants depending upon the material of the beam and its size. For a wrought-iron bar whose rectangular section is 2 in. deep and 1 in. wide, $I = .666$ and $E = 29 \times 10^6$.

Find the deflection of a 2 in. by 1 in. wrought-iron bar loaded with 4237 lb., the distance between the supports being 5 ft.

28. If $W = 5680$ lb., $L = 8$ ft. 4 in., $I = .846$ and $E = 32 \times 10^6$, find D , using the above formula.

N.	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2038	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3223	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3803	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

N.	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8063	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

CHAPTER IV.

THE APPLICATION OF ARITHMETIC TO MEASUREMENTS.

135. Tables. In the measurement of lengths, surfaces, and volumes, various *units of measure* are used. There are two systems of units used in this country: the *English* system, which is used in trade; and the *French* or *Metric* system, which is used in the sciences and by some departments of the United States government.*

The tables of these units are given here for review and for convenient reference.

136. English or common units.

UNITS OF LENGTH.

12 inches (in.)	=1 foot (ft.)
3 feet	=1 yard (yd.)
16½ feet, or 5½ yards	=1 rod (rd.)
320 rods, or 5280 feet	=1 mile (mi.)

UNITS OF SURFACE.

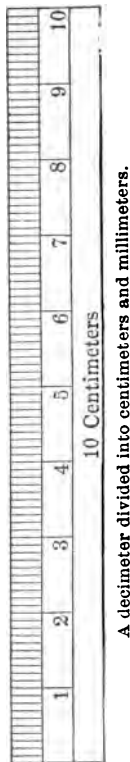
144 square inches (sq. in.)	=1 square foot (sq. ft.)
9 square feet	=1 square yard (sq. yd.)
30½ square yards	=1 square rod (sq. rd.)
160 square rods	=1 acre (A.)
640 acres	=1 square mile (sq. mi.)

* The metric system of measures was adopted by the French government in the early part of the nineteenth century. It is now used by most of the civilized nations, except the United States and Great Britain. On account of its superiority over the English system, still used by these two nations, it is hoped that they will join in making its use universal.

UNITS OF VOLUME.

1728 cubic inches (cu. in.)	=1 cubic foot (cu. ft.)
27 cubic feet	=1 cubic yard (cu. yd.)
128 cubic feet	=1 cord of wood (cd.)

137. The French or metric units. The *metric* system is a *decimal* system, the number of *units* of one denomination making *one unit* of the next higher being always a power of 10. It is this that makes the metric system superior to the other system. Thus, reduction from one denomination to another merely involves changing the position of the decimal point.



The *standard unit* of the metric system, from which all other units are derived, is the **meter**. It was obtained by surveying a long distance on the meridian through Paris and computing the distance from the equator to the pole of the earth and taking .0000001 of this distance.

A slight error was made in fixing the length of the meter, but that does not affect the usefulness of the system. A meter equals about 39.37 inches.

UNITS OF LENGTH.

1000 mikrons (μ)	=1 millimeter (mm.)
10 millimeters	=1 centimeter (cm.)=0.3937 inch.
10 centimeters	=1 decimeter (dm.)
10 decimeters	=1 meter (m.)=39.3707 inches.
10 meters	=1 dekameter (Dm.)
10 dekameters	=1 hektometer (Hm.)
10 hektometers	=1 kilometer (Km.)= $\left\{ \begin{array}{l} 3380.9 \text{ feet, or} \\ 0.6214 \text{ mile.} \end{array} \right.$
10 kilometers	=1 myriameter (Mm.)

NOTE.—In the tables, units in most common use are in bold-faced type.

UNITS OF SURFACE.

100 sq. millimeters (sq. mm.)	= 1 sq. centimeter (sq. cm.) = 0.155 sq. in.
100 sq. centimeters	= 1 sq. decimeter (sq. dm.)
100 sq. decimeters	= 1 sq. meter (sq. m.) = 10.764 sq. ft.
100 sq. meters	= 1 sq. dekameter (sq. Dm.)
100 sq. dekameters	= 1 sq. hektometer (sq. Hm.)
100 sq. hektometers	= 1 sq. kilometer (sq. Km.) = 247.114 acres.

When used in measuring land, the square meter is also called a **centare** (ca.), the square dekameter an **are** (a.), and the square hektometer a **hektare** (Ha.).

UNITS OF VOLUME.

1000 cu. millimeters (cu. mm.)	= 1 cu. centimeter (cu. cm.) = 0.06102 cu. in.
1000 cu. centimeters	= 1 cu. decimeter (cu. dm.)
1000 cu. decimeters	= 1 cu. meter (cu. m.) = 35.314 cu. ft.

In measuring wood, the cubic meter is also called a **stere**.

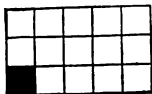
ORAL EXERCISES.

1. How many mm. in a m.? In a Km.?
2. How many cm. in a m.? In a Km.?
3. Reduce 25 Km. to m.
4. Reduce .03 μ to mm.
5. Reduce .37 Km. to m.
6. How many m. in 1.22 Dm.? In 2 Hm.? In .004 Km.?
7. How many cm. in 2.45 mm.? In 3 m.?
8. How many Hm. in 426.25 m.? How many cm.? How many Km.?
9. Reduce 4.25 sq. cm. to sq. m. To sq. mm.
10. How many sq. cm. in 25 sq. m.? In 4286 sq. mm.?
11. How many sq. m. in 36284 sq. mm.? In 12.34 sq. cm.?
12. Reduce .0286 sq. Km. to sq. Dm. To sq. m. To sq. dm. To sq. cm. To sq. mm.
13. How many cu. m. in 1 cu. cm.? In 1 cu. mm.?

14. How many cu. m. in 1 cu. Dm.? In 1 cu. Km.?
15. Reduce 26.814 cu. m. to dm. To cu. cm. To cu. cm.
16. Reduce 5.123 cu. m. to cu. Dm. To cu. cm.
17. How many a. in 5.268 Ha? How many ca.? How many sq. m.?
18. How many cu. dm. in 12 steres? How many cu. Dm.?
19. How many feet in 2.5 m.? In 327 cm.?
20. How many sq. in. in 100 sq. cm.? In 326.8 sq. mm.?
21. How many in. in 2.5 cm.? In 500 mm.?
22. How many miles in 15 Km.?
23. The regulations of the United States government regarding foreign mail require that packages of merchandise sent through the mail may not exceed 30 cm. in length, 20 cm. in width, and 10 cm. in height. Express these dimensions in inches.

138. Geometrical measurements. The geometrical measurements discussed in the following paragraphs are used very commonly in everyday life, in business, in technical pursuits, and in science. Rigorous proofs of the principles involved can not be given here, but will be found in geometry. Students who have not had geometry should find the explanations here given sufficiently satisfying.

139. Area of a rectangle. If a rectangle is 5 in. long and 3 in. wide, it may be divided into squares, as in the figure, each square being 1 sq.in. One horizontal strip contains 5 sq.in., and since there are 3 strips, the area of the whole rectangle is 3×5 sq.in., or 15 sq.in.



In general, the area of a rectangle b in. long and w in. wide is $w \times b$ sq.in., or wb sq.in. The sides of a rectangle are called the **base** and **altitude**. This principle is usually expressed thus:

The area of a rectangle equals the product of its base and altitude.

NOTE.—It is clear that what is meant in the statement of this principle is that the *number of units* of surface equals the product of the *number of units of length* in the base and the *number of units of length* in the altitude. It is impossible to take the *product of two lines, as such*. What we do multiply are the abstract numbers. The same remark applies to the other principles which follow in this chapter.

140. Area of a square. Since a square is a rectangle whose base and altitude are equal, from the above principle it follows that

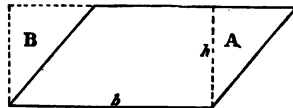
The area of a square equals the square of one side.

From this it follows that

The length of a side of a square may be obtained by extracting the square root of the numerical value of its area.

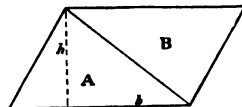
Thus, if the area of a square lot is 2704 sq. m., the length of one side is $\sqrt{2704}$ m., or 52 m.

141. Area of any parallelogram. The area of any parallelogram is the same as that of a rectangle having an equal base and an equal altitude, as is shown by cutting off the triangle A from one end of the parallelogram and placing it in the position B at the other end, thus converting it into a rectangle. Hence:



The area of any parallelogram equals the product of its base and altitude.

142. Area of a triangle. Since a second triangle congruent to the given triangle A may be placed in the position B, the two thus forming a parallelogram with the same base and altitude as the triangle A, it follows that



The area of a triangle equals one-half the product of its base and altitude.

A second method of computing the area of a triangle may be used when the length of the altitude is not known, but the lengths of all three sides are given. It is proved in geometry that

The area of a triangle equals the square root of the product of the half-sum of the sides and the three remainders obtained by subtracting from this half-sum each of the sides in succession.

EXAMPLE. Find the area of the triangle whose sides are 12.5 cm., 15.3 cm., and 20.4 cm., respectively.

The half-sum is 24.1 cm., and the remainders are 11.6 cm., 8.8 cm., 3.7 cm.

Hence we are to find $\sqrt{24.1 \times 11.6 \times 8.8 \times 3.7}$ sq. cm.

$$\log 24.1 = 1.3820$$

$$\log 11.6 = 1.0645$$

$$\log 8.8 = .9445$$

$$\log 3.7 = .5682$$

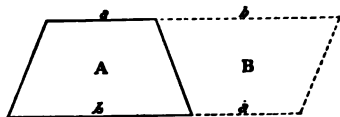
$$\begin{array}{r} 2)3.9592 \\ 1.9796 \end{array}$$

$$1.9796 = \log 95.42$$

Therefore, the area is 95.42 sq. cm.

NOTE.—Students who have not had logarithms should perform the multiplication and extract the square root.

143. Area of a trapezoid. A second trapezoid, congruent to the given trapezoid A, may be placed in the position B, so



that the two form a parallelogram. The area of the trapezoid A equals one-half the area of the parallelogram. Since the parallelogram has the same altitude as the trapezoid, and its

base is the sum of the bases of the trapezoid, it follows that

The area of a trapezoid equals one-half the product of the altitude and the sum of the bases, or the product of the altitude and one-half of the sum of the bases.

144. Area of any polygon. The area of any polygon may be obtained by dividing it into triangles, or any other figures whose areas may be found, then finding the areas of these parts and adding them.

PROBLEMS.

1. How many yards of 36-inch carpet are required to cover a floor 12 feet wide and 14 feet long, no allowance being made for matching?

2. How many strips, running lengthwise, of Brussels carpet (27 in. wide) are required for a room 15 ft. by 20 ft.? How many running yards in each strip? How many running yards in all, if there is no waste in matching? What is the cost at \$1.25 per yard?

3. A rectangular piece of land is 40 rods wide and 55 rods long. Find the area in square rods; in acres.

4. A city block is 400 feet wide and 600 feet long. How many acres does it cover?

5. How much does it cost to construct a concrete sidewalk 5 ft. wide and 164 ft. long at \$1.25 per sq. yd.?

6. What does it cost to pave a street 42 ft. wide and 400 ft. long at \$1.62 per sq. yd.?

7. How many bricks 8 in. long and 4 in. wide are required to lay a sidewalk 6 ft. 8 in. wide and 64 ft. long?

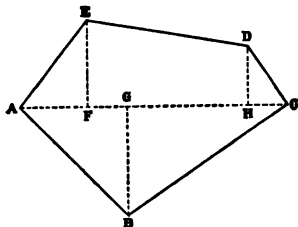
8. What does it cost to lath and plaster a room 14 ft. by 16 ft. and 10 ft. high, at 33 cents per sq. yd., no allowance being made for openings?

9. How many tiles 8 in. square will it take to cover a court 33 ft. 4 in. long and 16 ft. 8 in. wide?

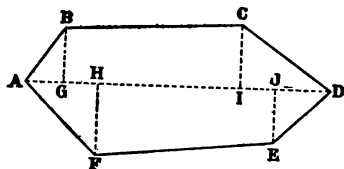
10. A square park contains 1000 acres. How many yards of iron fence does it take to enclose it?

11. A rug cleaning establishment charges 12 cents a yard for cleaning carpet 27 inches wide. At the same rate per yard, what will it cost to clean two rugs, one 7 ft. 6 in. by 5 ft. 5 in., the other 5 ft. 8 in. by 3 ft. 6 in.?

12. How many barrels of lime will be required to plaster the walls and ceiling of a hall 40 ft. long, 10 ft. 6 in. high and 22 ft. wide if it takes $3\frac{1}{4}$ bbl. of lime for 100 sq. yd.?

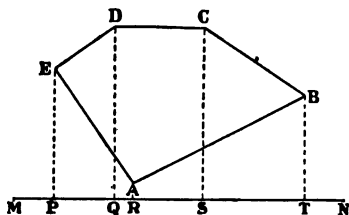


13. Find the area of the polygon $ABCDE$, if $AC=500$ yd.; $BG=200$ yd.; $AF=75$ yd.; $AG=125$ yd.; $AH=425$ yd.; $EF=160$ yd.; $DH=110$ yd.



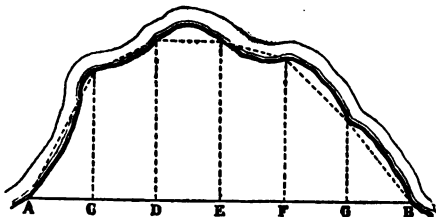
14. In the polygon $ABCDEF$, $AG=100$ ft.; $GH=100$ ft.; $HI=400$ ft.; $IJ=100$ ft.; $JD=150$ ft.; $BG=150$ ft.; $HF=200$ ft.; $CI=175$ ft.; and $JE=150$ ft. Find the area.

15. Surveyors may determine the amount of land in a given tract as follows: A base line MN is staked off east and west, say. The distances are



The distances are measured at right angles from various points in the boundary of the land, as A , B , C , etc., meeting the base line at P , Q , R , etc. The distances PQ , etc., are measured. Show how the area of the land may be computed.

16. A farmer sold to his neighbor a strip of land, lying between the road AB and the river, for \$60 per acre. He had it surveyed. The surveyor measured along the road AB, driving a stake every 100 ft., at C, D, etc., finding AB to be just 600 ft. From each of these stakes he measured the distance, at right angles to AB, to the river. He found these distances to be: from C, 200 ft.; from D, 250 ft.; from E, 250 ft.; from F, 225 ft.; from G, 125 ft. The area was then computed. How much did the farmer get for the land?



17. A gallon of paint costing \$1.50 covers 300 sq. ft. with one coat. Find the cost of painting a tight board fence with one coat on each side if the fence is 4 ft. 6 in. high, and encloses a lot 21 rd. long and 9 rd. wide.

18. How many sq. ft. of 1-inch pine boarding will be required to make 1,000 packing cases, each 27 in. long, 18 in. wide, 12 in. deep, allowing 10% waste on each box?

19. A rectangular grass plot is 110 ft. by 90 ft. Around it is a gravel path 4 ft. wide. A contractor agrees to cover the path with asphalt at 5 cents per sq. ft. What does the contract cost?

20. How many square feet of sheathing are necessary for the outside, including top, of a freight car 34 ft. long, 8 ft. wide, and $7\frac{1}{2}$ ft. high, allowing 10% for waste?

21. A planing-machine cuts $\frac{1}{2}$ in. wide. How long will it take to plane a casting 20 ft. by 10 ft. 6 in., if the work moves at 20 ft. per minute?

22. The pedestal of a marble column is in the form of a frustum of a pyramid whose bases are regular octagons with sides 3 ft. and 2 ft. 8 in., respectively, and whose slant height is 1 ft. 2 in. How much surface must be polished?

23. A monument of marble consists of a frustum of a square pyramid whose bases are 2 ft. and 1 ft., respectively, and slant height 5 ft., surmounted by a pyramid whose slant height is 1 ft. What is the cost of polishing the surface at 60 cents per sq. ft.?

24. A roof has 4 sides; two in the form of trapezoids with bases 30 ft. and 10 ft., and altitude 15 ft.; and two in the form of triangles with base 20 ft. and altitude 15 ft. If 1000 shingles cover 120 sq. ft., how many thousand are required to cover the roof?

25. Cement sidewalks are usually made as follows: an excavation 4 inches below the surface of the proposed walk is made and thoroughly rammed. Then a 3-inch layer of cement concrete is spread. Upon this base, a 1-inch wearing surface is spread. The concrete is 1 part cement to 7 parts gravel and sand, and the wearing surface is 1 part cement to 1 part sand. The cement costs \$1.80 and the sand and gravel 10 cents per barrel. How much per sq. ft. must a contractor get for furnishing the walks for a city, if the labor costs 125% of the material and he wishes to make 20% of the net cost (material and labor) of laying the walks? (1 bbl. = 2.84 cu. ft.)

26. In calculating the cost of lathing and plastering a room, the total area of wall space is considered without any deductions for doors and windows.

Make the following estimates for a room $9\frac{1}{2}$ feet high, 16 feet wide, and 22 feet long:

Laths are worth \$6 per 1000 and are put up in bunches of 50. A bunch will lath 4 sq. yd. How many laths are required? What will they cost? (A part of a bunch cannot be bought.)

Hard wall-plaster is worth \$6.50 per ton and is put up in 100 lb. bags. A ton will plaster 100 sq. yd. How much plaster will it take? What will it cost? (A part of a bag cannot be bought.)

How much does the workman get for his work if he charges 33 cents per square yard, including material, to complete the job?

145. Relation of the circumference to the diameter of a circle.

If the circumferences and diameters of several circular objects, such as circular discs cut from stiff cardboard, plates, wheels, or coins, are measured,* and the circumference of each divided by its diameter, then the average of these quotients computed to several decimal places, the average will be found to equal approximately 3.1416. Hence,

The circumference of a circle equals approximately 3.1416 times the diameter.

The number 3.1416 is only the approximate value of an incommensurable number 3.1415927 This has been computed to over 700 decimal places.† The value 3.1416 will give very accurate results in all practical problems where it is used. This incommensurable number, the quotient of the circumference divided by the diameter of the circle, is usually represented by the Greek letter π (pi).

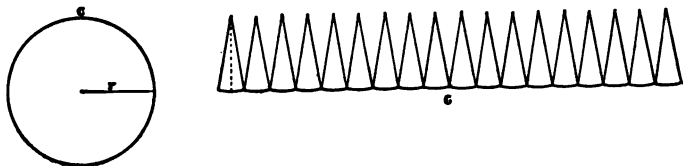
If c stands for the circumference, d the diameter, and r the radius of a circle, we have

$$c = \pi d = 2\pi r.$$

* To get an accurate measurement of a circumference, two pupils can work together to an advantage. Take two rulers, one standing edge-wise on the other as a guide for the circle. Mark a point on the circumference and roll through one complete revolution, noticing the distance passed over on the bottom ruler. Hold the circle against the second ruler to get a straight path.

† To 96 places, the value of π is 3.141592653589793238462643383279502884197169399875105820974944592307816406286208998628034825342117.

146. Area of a circle. A circle with radius r and circumference c can be cut along a large number of radii into equal figures which resemble triangles. If these were triangles, the altitude of each would be r and the sum of the bases, the circumference c .



Considering them triangles, the area would be that of a single triangle whose base is the circumference, and whose altitude is the radius. This is proved in geometry to be the exact area. Hence,

The area of a circle equals one-half the product of the circumference and radius.

Since by § 145, $c = 2\pi r$, the area equals $\frac{1}{2} \times 2\pi r \times r$, or πr^2 . Hence,

The area of a circle equals π times the square of the radius.

EXAMPLE 1. Find the area of the circle whose radius is 4 inches.

The area $= 3.1416 \times 4^2$ sq. in. $= 50.2656$ sq. in.

EXAMPLE 2. Find the radius of the circle whose area is 125 sq. cm.

$$\text{The radius} = \sqrt{\frac{125}{3.1416}} \text{ cm.}$$

$$\log 125 = 2.0969$$

$$\log 3.1416 = 0.4971$$

$$\begin{array}{r} 2)1.5998 \\ \hline \end{array}$$

$$.7999 = \log 6.308$$

Hence, the radius is 6.308 cm., approximately.

NOTE.—Let the student who has not had logarithms perform the division and extract the square root.

PROBLEMS.

1. Find the diameter of a wheel that makes 660 revolutions in going a mile.

2. How many revolutions per mile will a 28-inch bicycle wheel make?

3. What is the area of a circular ring that is 3 inches in inside diameter and 4 inches in outside diameter?

4. The circular basin of a fountain is 20 ft. in diameter. What will it cost to cement the floor of it at \$1.75 per sq. yd.?

5. A boiler of an engine has 300 tubes, each 3 inches in inside diameter. What is the total cross-sectional area?

6. A cylindrical tank is 18 ft. in diameter. How long must a piece of strap iron be taken to make a band around it, allowing 1 ft. for overlapping?

7. The earth is nearly 8000 miles in diameter. What is the approximate length of the equator? At what velocity does a person who is at the equator revolve through space?

8. A cylindrical tin can is to be made 4 inches in diameter and $4\frac{1}{2}$ inches high. Find the dimensions of the piece of tin that is to form the cylindrical surface, $\frac{1}{4}$ inch being allowed for a seam.

9. How much belting does it require to make a belt to run over two pulleys, each 30 in. in diameter, the distance between their centers being 18 ft.?

10. A belt runs over two pulleys, one being 4 ft. in diameter and driven by an engine at 100 revolutions a minute. What must be the diameter of the other pulley, if it is to turn a fan at the rate of 400 revolutions a minute?

11. If a 10-inch pulley is making 300 revolutions a minute, how fast is the rim traveling in feet per minute?

12. A grindstone of Ohio stone will safely stand a surface

speed of 2500 feet per minute. How many revolutions per minute will a 4-ft. stone safely stand? How many revolutions per minute will a 3-ft. stone stand? A $5\frac{1}{2}$ -ft. stone?

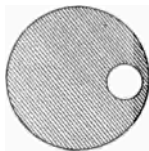
13. Grindstones of Huron stone will stand a surface speed of 3600 feet per minute. How many revolutions per minute will a $3\frac{1}{2}$ -ft. stone stand? A 6-ft. stone?

14. An emery stone will safely stand a surface speed of 5500 ft. per minute. If the spindle of an emery grinder makes 1800 revolutions per minute, what is the diameter of the largest wheel that may safely be used?

15. A 10-inch pulley runs 300 revolutions per minute and is to drive a machine at the rate of 180 revolutions per minute. What size of pulley must be on the machine?

16. The main driving pulley of an engine is 12 ft. 6 in. in diameter and makes 96 revolutions per minute. It is belted to a 48-inch pulley on the main line shaft. Find the speed of the latter.

17. When there is a steam pressure of 90 pounds per square inch, what is the pressure on a 12-inch piston? On a 6-inch piston?



18. The cross-sectional diameter of a shaft is 8 inches. In this is a 2-inch hole eccentric to the shaft. Find the area of the cross section.

19. A running track having two parallel sides and two semicircular ends, each equal to one of the parallel sides, measures exactly a mile at the inner curb. Two athletes run, one five feet from the inner curb and the other ten feet from it. By how much is the second man handicapped?

20. A boy throws stones with a sling, the sling describing a circle with a radius of 4 ft. If the string is whirled at the

rate of 3 revolutions a second, at what velocity does a stone leave it?

21. A well is 40 ft. deep and the axle on which the chain is coiled is $7\frac{1}{2}$ in. in diameter. How many turns of the handle are necessary to raise the bucket from the bottom to the surface?

22. The carrying capacity of 9 pipes, each 3 in. in diameter, is the same, neglecting friction, as that of one pipe with what diameter?

23. What is the relation between the carrying capacity of one 6-inch pipe and two 3-inch pipes when each pipe discharges water with a velocity of 80 ft. per minute?

24. How many 2-in. flues of an engine will equal in sectional area a smokestack 22 in. in diameter?

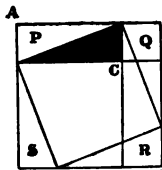
25. A horizontal boiler has 60 tubes, each 2 in. in diameter. What must be the diameter of the smokestack, if the rule requires one whose sectional area equals the combined sectional areas of the tubes?

26. If, in iron turning, it is customary to allow a cutting speed at the rim of 40 ft. per minute, at what speed (revolutions per minute) ought the lathe be set for a piece of iron 3 in. in diameter?

147. The theorem of Pythagoras.* This principle is that

The square on the hypotenuse of a right-angled triangle equals the sum of the squares on the other two sides.

* This principle is called the *Theorem of Pythagoras* after the Greek philosopher Pythagoras who is said to have proved it about 500 B.C. It was known, for special cases at least, by the Egyptians, who are believed to have used it in building the pyramids a long time before the time of Pythagoras. He is the first man believed to have given a rigorous proof for it. Over seventy proofs have now been worked out for it.



The truth of it may be seen by reference to the figure. If the four triangles P, Q, R, S, are taken away from the whole figure, the square on the hypotenuse of the shaded right-angled triangle remains. But these triangles together are evidently equivalent to the sum of the rectangles AC and CB. And if these rectangles are taken away from the whole figure, the sum of the squares on the other two sides of the right-angled triangle remains, which proves the theorem.

This theorem is one of the most useful principles of elementary mathematics, especially in the measurements of distances.

EXAMPLE 1. The base and altitude of a right-angled triangle are 3 m. and 4 m., respectively. Find the length of the hypotenuse.

The square on the base = 9 sq. m.

The square on the altitude = 16 sq. m.

Hence, the square on the hypotenuse = 9 sq. m. + 16 sq. m., or 25 sq. m.

Therefore, the hypotenuse = $\sqrt{25}$ m., or 5 m.

EXAMPLE 2. If the hypotenuse of a right-angled triangle is 64 ft. and one side 50 ft., what is the length of the other side?

The other side = $\sqrt{64^2 - 50^2}$ ft., or 39.9 ft., approximately.

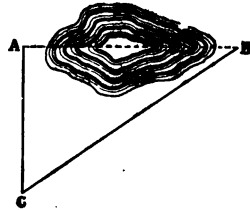
PROBLEMS.

1. The sides including the right angle of a right-angled triangle are 5 m. and 12 m. respectively. Find the hypotenuse.
2. The hypotenuse and one side of a right-angled triangle are 100 ft. and 64 ft., respectively. Find the other side.
3. A baseball diamond is a square 90 ft. on the side. What is the distance from first to third base?
4. How long is the diagonal of a rectangle which is 48 ft. long and 42 ft. wide?
5. Can a wheel 8.1 ft. in diameter be taken through a door 7 ft. high and 4 ft. wide?
6. If each of the three sides of a triangle is 30 cm., how long is the altitude? What is the area?

7. A derrick is 48 ft. high, and is supported by three cables, each reaching from the top of the derrick to a stake in the ground 45 ft. from the foot of the derrick. How much steel cable does it take, allowing 10 ft. for fastenings?

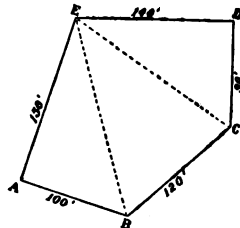
8. A gable-roof house is 24 feet wide. The distance from the plate to the ridge-pole is to be 12 feet. How long must the carpenter cut the rafters, if they are to project 1 foot over the eaves?

9. A surveyor, wishing to determine the distance between two points A and B which are separated by a lake, measures a line AC 250 yards long at right angles to AB, and then the distance from C to B. CB is found to be 325 yards. What is the distance from A to B?



10. It is desired to determine the distance between two points A and B on opposite sides of a hill. A third point C is located from which the directions to A and B form a right angle, and the distances CA and CB are measured. If $CA = 400$ yd., and $CB = 720$ yd., find the distance between A and B.

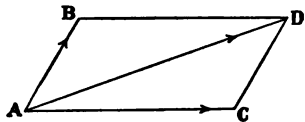
11. The figure is a diagram of a polygon whose angles at A and D are right angles. The sides are $AB = 100$ ft.; $BC = 120$ ft.; $CD = 90$ ft.; $DE = 140$ ft.; $EA = 150$ ft. Find the area.



SUGGESTION. Find the lengths of the diagonals EB and EC. Then find the areas of the three triangles.

Two forces exerted upon an object at A, one represented in

magnitude and direction by the line AB , and the other by the line AC , have the same effect as a single force, called their resultant, which is represented in magnitude and direction by the diagonal AD of the parallelogram $ABCD$.



12. If two forces, one of 75 lb. and the other 100 lb., are exerted upon an object at right angles to each other, what is their resultant?

13. One of two forces which act at right angles is 12 lb., and their resultant is 15 lb. What is the other force?

14. A man swims across a river at the speed of 5 ft. per second, and the current carries him down stream at the rate of 3 ft. per second. What is his resultant speed through the water? If the stream is 200 yards wide, at what point will he reach the opposite bank?

15. Find the distance between the lower corner and the upper opposite corner of a room 40 ft. long, 24 ft. wide, and 16 ft. high.

16. The sides of a right-angled triangle are equal, and the hypotenuse is 15 cm. What is the length of the equal sides? What is the area?

17. At 45° north latitude, what is the surface speed of points on the surface of the earth in miles per hour, considering the radius of the earth to be 4000 miles?

SUGGESTION. The place moves in a circle whose radius is at right angles with the axis of the earth.

18. How much longer is the side of a square circumscribing a circle 4 ft. in diameter than the side of a square inscribed in the circle?

19. In making an Indian Club, pieces of holly were glued to the faces of a piece of gum wood 3 inches square. This was

turned in a lathe until the holly produced oblong, oval light spots on the surface of the club just touching at the point of greatest thickness. What must have been the thickness of the holly to avoid waste?

20. To ream round holes 4 inches in diameter a square reamer is used. Find the dimensions of the reamer. (The diagonal is that which reams the holes.)

148. Measurement of timber. Practical foresters, lumbermen, and others have occasion to compute the amounts of timber of certain kinds in forests. In practice, to compute the amount of lumber that a given tree will produce, lumbermen make use of fixed rules and tables of values. There are over forty of these rules in use in different parts of the country. For a full discussion of forest mensuration the student is referred to special books on the subject.*

149. Board measure. In measuring lumber, the unit of measure used is the **board foot**. A board foot is the measure of a board 1 ft. long, 1 ft. wide, and 1 in. thick. In measuring boards less than an inch thick, the computation is the same as if the boards were an inch thick.

Thus, a board 1 in. thick, 2 ft. wide, and 16 ft. long, contains 32 board feet. A board $\frac{1}{2}$ in. thick, 12 in. wide, and 10 ft. long, contains 10 board feet. A joist 3 in. thick, 5 in. wide, and 12 ft. long, can be divided into 3 pieces, each 1 in. thick, and therefore contains $3 \times \frac{1}{12} \times 12$ board feet, or 15 board feet.

A board foot is usually called simply a *foot*. Lumber is usually bought and sold by the 1000 feet.

Thus "\$8 per M" means \$8 per thousand board feet.

ORAL EXERCISES.

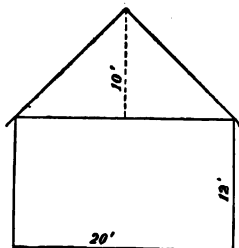
How many board feet in each of the following:

1. A board 1 in. thick, 1 ft. wide, and 8 ft. long?

* See "The Woodman's Handbook," by Henry Solon Graves, printed by the Government Printing Office, Washington, D. C.

2. A board 1 in. thick, 8 in. wide, and 12 ft. long?
3. A board 1 in. thick, 6 in. wide, and 9 ft. long?
4. A board $\frac{1}{2}$ in. thick, 12 in. wide, and 3 ft. long?
5. A board $\frac{5}{8}$ in. thick, 9 in. wide, and 12 ft. long?
6. A board 2 in. thick, 1 ft. wide, and 10 ft. long?
7. A joist 3 in. by 4 in., and 16 ft. long?
8. A joist 2 in. by 6 in., and 16 ft. long?
9. A beam 4 in. by 8 in., and 14 ft. long?
10. A plank $1\frac{1}{2}$ in. by 10 in., and 12 ft. long?
11. Eight "3 by 4" joists, each 15 ft. long?
12. Twelve joists, each 16 ft. long and 4 in. square?
13. A stick of timber 18 ft. long and 12 in. square?

PROBLEMS.



1. The diagram gives the dimensions of the gable end of a house. Allowing for a window space of 5×6 sq. ft., how many feet of lumber does it take to "rough side" it? What does it cost at \$18 per M?
2. Regular siding, or "weather boarding," is 5 inches wide, but being overlapped, lies but $3\frac{3}{4}$ inches to the weather. Hence $\frac{1}{8}$ is added to the number of square feet in estimating the amount of siding needed for a house. What will regular siding cost at \$28 per M for the surface described in Ex. 1?
3. How much lumber is required to floor a barn 40 ft. long and 24 ft. wide with 2-inch boards?
4. How much 1-inch boarding will it take to build a board fence 4 ft. high and 166 ft. long between two lots? What will it cost at \$25 per M?

5. A contractor, in building a house, bought lumber as follows:

- 3000 ft. of rough siding at \$22 per M;
- 4000 ft. of weather boarding at \$28 per M;
- 1800 ft. of flooring at \$60 per M;
- 700 ft. for inside finishing at \$35 per M;
- 200 "3 by 4" joists, each 12 ft. long, at \$25 per M.

What was the cost?

6. Hard wood flooring is quite commonly 3 inches wide with a $\frac{3}{4}$ -inch "tongue and groove." How wide a strip will one board cover? How much must be added to the area of a floor to allow for the "tongue and groove" of 3-inch flooring?

7. Find the cost of 3-inch oak flooring at \$60 per M for three rooms as follows: $12' \times 14'$; $15' \times 16'$; and $18' \times 20'$. (Add $\frac{1}{3}$ of the area for waste in the tongue and groove.)

8. A carpenter adds 50% of the cost of material for laying and cleaning the floors, and the painter charges 35 cents per square yard for filling and waxing. Find the total cost of the floors described in Example 7.

9. All flooring has the $\frac{3}{4}$ -inch tongue and groove. What allowance must be made for 6-inch flooring; *i. e.*, what must be added to the surface of the floor to find the board feet of lumber needed?

10. Find the cost of flooring for four bedrooms as follows:

$11' \times 12'$; $10\frac{1}{2}' \times 14'$; $12' \times 13\frac{1}{2}'$; $14' \times 16'$.

Use 6-inch pine flooring (add $\frac{1}{7}$) at \$24 per M, and add 35% of the cost of material for the work of laying it.

150. The "Doyle Rule," which is most extensively used throughout the country for computing the contents in board feet of a log, is as follows:

Subtract 4 inches from the diameter of the log at the small

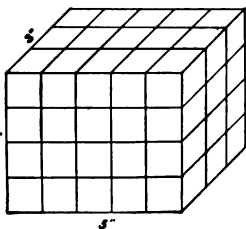
end; square one-quarter of the remainder; and multiply the result by the length of the log in feet.

EXERCISES.

Fill out the following chart to show the board feet of lumber in logs as follows:

Diameter in inches	12	13	14	15	16	17	18	19	20	24
Length in feet	10	12	14	16	18	20	16	18	24	24
Number of board ft										

151. Volume of a rectangular parallelopiped.



A rectangular parallelopiped 5 in. long, 4 in. high, and 3 in. wide, as in the figure, may be cut into inch cubes. There will be 4 layers of cubes, with 3 rows in each layer, and 5 cubes in each row. Hence the volume, or total number of cubic inches, is $4 \times 3 \times 5$ cu. in., or 60 cu. in.

In general, the volume of a rectangular parallelopiped with length l in., width w in., and height h in., is $l \times w \times h$ cu. in., or lwh cu. in.

The volume of a rectangular parallelopiped equals the product of its three dimensions.

Since the product of the length by the width equals the area of the base, this rule may be expressed thus: (See § 139, Note.)

The volume of a rectangular parallelopiped equals the product of the altitude and the area of the base.

NOTE. It is not important that it be shown in this course, but it is proved that

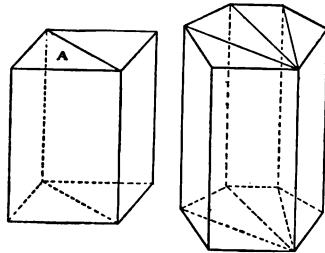
The volume of any parallelopiped equals the product of the altitude and the area of the base.

152. Volume of a prism.

The volume of any triangular

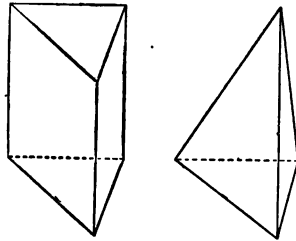
prism, as prism A, equals half the volume of a parallelopiped with the same altitude and double the base. Hence, the volume of the prism A equals that of a parallelopiped with the same altitude and equal base.

Any prism may be divided into triangular prisms, as in the figure. Hence, it is clear that the volume of any prism equals that of a parallelopiped with equal altitude and equal base. Therefore,



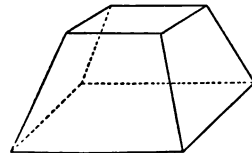
The volume of any prism equals the product of the altitude and the area of the base.

153. Volume of a pyramid. It is proved in geometry that the volume of any pyramid equals one-third of that of a prism with equal base and equal altitude. This may be verified as follows: Make from cardboard a hollow pyramid and a hollow prism with equal base and equal altitude. Fill the pyramid three times with sand, and pour the contents into the prism. The prism will be exactly filled. Therefore,



The volume of any pyramid equals one-third of the product of the altitude and the area of the base.

154. Volume of a frustum of a pyramid. The frustum of a pyramid is that portion of the pyramid contained between the base and a plane parallel to the base. It is shown in geometry that the volume of a frustum of a pyramid may be obtained by the rule:



To the sum of the bases, add the square root of their product, and multiply this sum by one-third the altitude.

E. g., the volume of a frustum whose height is 6 ft., area of lower base 27 sq. ft., and area of upper base 12 sq. ft., is

$$\frac{1}{3} \times 6 \times (27+12+\sqrt{27 \times 12}) \text{ cu. ft., or } 114 \text{ cu. ft.}$$

PROBLEMS.

1. The length of a box is 25 cm., width 20 cm., and volume 8000 cu. cm. Find the depth.

2. The wagon box most generally used in hauling dirt is 9 ft. by 3 ft. by 16 inches. How much dirt will a load contain?

3. A square vat is 3 m. deep, and it holds 75 cu. m. Find its length and width.

4. A freight car is 36 ft. long, and 8 ft. 6 in. wide, inside measure. How many bushels of wheat will it contain if loaded to a depth of 5 ft.? What is the weight of the car of wheat? (A cubic foot is approximately .8 bu. 1 bu. of wheat weighs 60 lb.)

5. What is the edge of a cube whose volume is 830.584 cu. cm.?

6. The excavation for a house is to be 40 ft. long, 28 ft. wide, and 6 ft. deep. How many cubic yards (loads) of earth must be removed? What is the cost at 35 cents per cu. yd.?

7. A stone-mason furnishes the material and lays a front wall 42 feet long, 8 feet high and 16 inches thick at \$5.75 per cu. yd. Find the cost.

8. A stone-mason builds 4 pillars, each 3 ft. by 3 ft. by $7\frac{1}{2}$ feet high, and a front wall 36 feet long, 3 feet high and 18 inches thick, charging \$25 per cord (128 cu. ft.) for the work and material. Find the total cost.

9. How many cords of wood in a rick 4 ft. wide, 16 ft. long, and 6 ft. high?

10. How many solid cords of 16-inch wood can be ricked in a room 20 ft. long, 18 ft. wide, and 12 ft. high?

11. A tank 4 ft. 2 in. long, 3 ft. wide, and 5 ft. 6 in. high holds how many gallons? (A gallon = 231 cu. in.)

12. How many tons of ice can be packed in an ice-house 40 ft. by 24 ft., and 16 ft. high, allowing 2 ft. on each side and above and below for sawdust? (1 cu. ft. weighs 56.25 lb.)

13. How many square feet of ice must be cut to fill a car 34 ft. long, $8\frac{1}{2}$ ft. wide, 5 layers deep? What will the car of ice weigh if it is 14 inches thick?

14. An ice-box will hold a piece of ice 32 inches long, 24 inches wide and 14 inches thick. What will such a piece weigh?

15. A prism has an altitude of 6 in., and its base is a triangle whose sides are each equal to 4 in. What is the volume?

16. A cistern in the form of a regular hexagonal prism (the base has 6 equal sides and equal angles) is 6 ft. on its basal edges and 7 ft. deep. How many gallons does it hold?

SUGGESTION. The base may be divided into 6 equal triangles with their sides all equal.

17. How many cubic meters of water could a V-shaped gutter discharge (flowing full) in a day, if its depth is 20 cm., its sides meet at right angles, and the water flows a meter per second?

18. In constructing a railroad, a cut is made 373 ft. long, 12 ft. deep, 30 ft. wide at the bottom, and 40 ft. wide at the top. How many cubic yards of excavation are made?

19. Find how many tons of coal a bin 20 ft. by 16 ft. by 8 ft. will hold, allowing 80 cu. ft. to a ton.

20. How many bricks 2 in. by 4 in. by 8 in. are required to build a wall 12 in. thick, 4 ft. high, and 20 ft. long, allowing 10% for space occupied by mortar?

21. How much stone is required to construct a dam 300 ft. long, 15 ft. high, 10 ft. wide at the bottom and 4 ft. wide at the top?

22. One of "Cleopatra's Needles," quarried in one piece, floated down the Nile, and erected at Heliopolis, Egypt, about 1500 B. C., is now in Central Park, New York. It is in the form of a frustum of a quadrangular pyramid 64 ft. high, 8 ft. square at the base, and 5 ft. square at the top, surmounted by a pyramid 7 ft. high. This stone is said to weigh 170 lb. per cu. ft. Compute the weight of the stone, and thus see what gigantic feats were performed in its removal and erection.

23. A factory chimney is to be constructed in the form of a frustum of a pyramid with a square base. The height is to be 150 ft., and the sides of the bases are to be 10 ft. and 16 ft. respectively. The flue is 6 ft. square throughout the entire height. How many thousands of bricks 2 in. by 4 in. by 8 in. will it require, allowing 10% for mortar?

24. A bar of iron whose cross-section is 4 in. square is forged into a bar 24 in. long and with a cross-section $2\frac{1}{2}$ in. by $1\frac{1}{2}$ in. What length of bar must be used?

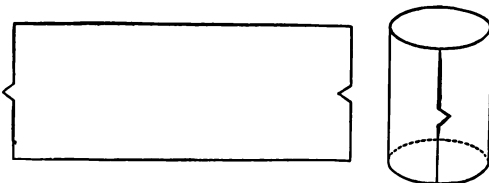
25. A bar of zinc 28 in. long, $2\frac{1}{2}$ in. wide, and $1\frac{1}{2}$ in. thick is melted into the form of a cube. What is the edge of the cube?

26. In a town lot 120 ft. long and 70 ft. wide, a cellar is to be dug for a building. The excavation is to be 42 ft. long, 36 ft. wide, and 8 ft. deep. The earth removed is to be used to "fill" the surrounding yard. What will be the depth of the filling?

27. Find the number of cubic yards of filling needed for a railroad embankment 400 ft. long, when its cross-section is a trapezoid 10 ft. high, 14 ft. on its lower base, and 8 ft. on its upper base.

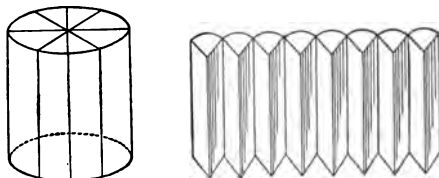
28. A cu. ft. of iron weighs 450 lb. How many pounds must be melted to make a hollow casting with 1-inch walls, the extreme dimensions of the casting being $28'' \times 16'' \times 12''$?

155. Surface of a cylinder. A rectangular piece of paper may be rolled into the form of the lateral surface of a right circular cylinder. Hence, it is evident that



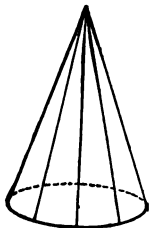
The lateral area of a right circular cylinder equals the product of the altitude by the circumference of the base.

156. Volume of a cylinder. A cylinder may be cut into wedge-shaped pieces resembling prisms, as in the figure. These prism-like solids have the same altitude as the cylinder, and the sum of their bases is equal to the base of the cylinder. If the pieces were prisms, the volume would be that of a single prism whose base equals the base of the cylinder and whose altitude equals the altitude of the cylinder. This is proved in geometry to be the exact volume. Hence,



The volume of a cylinder equals the product of the altitude by the area of the base.

157. Surface of a cone. A sector of a circle may be bent



into a right circular cone. By the same course of reasoning as that used in § 146, it may be seen that

The lateral area of a right circular cone equals one-half the product of the slant height by the circumference of the base.

158. Volume of a cone. By dividing a cone into parts that resemble pyramids, it is easily seen, by the method of reasoning used in § 156, that

The volume of a cone equals one-third of the product of the altitude by the area of the base.

159. Surface of a sphere. When a sphere is divided by a plane into two hemispheres, the plane surfaces thus exposed are *great circles* of the sphere.



They have the same radius as the sphere. The relation between the surfaces of a hemisphere and a great circle may be inferred from the following experiment:

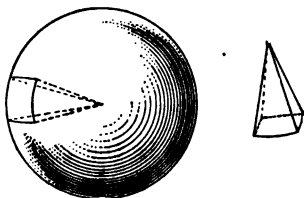
With a hard cord wind the surface of a hemisphere and of a great circle. If carefully done it will be found that it requires just twice as much cord to wind the hemisphere as to wind the circle. Hence, it will require four times as much to wind the surface of the whole sphere as to wind the circle. It is proved in geometry that

The area of the surface of a sphere equals four times the area of a great circle.

Since the area of a circle with radius r equals πr^2 , the area of the surface of a sphere with radius r equals $4\pi r^2$.

EXAMPLE. The area of the surface of a sphere with radius 5 in. is $4 \times 8.1416 \times 5^2$ sq. in., or 814.16 sq. in.

160. Volume of a sphere. By drawing three or more planes through the centre of a sphere, a portion is cut out, which resembles a pyramid with altitude equal to the radius. The entire sphere may be thus divided into a number of such solids, the sum of the bases of which is the surface of the sphere and the common altitude of which is the radius. If these parts were pyramids, we see that



The volume of a sphere equals the volume of a pyramid whose base is the surface of the sphere and whose altitude is the radius.

This is proved in geometry to be the exact volume.

Since the area of the surface is $4\pi r^2$, the volume is $\frac{1}{3} \times r \times 4\pi r^2$, or $\frac{4}{3}\pi r^3$.

EXAMPLE. The volume of a sphere whose radius is 9 cm. equals $\frac{4}{3} \times 3.1416 \times 9^3$ cu. cm., or 3053.6352 cu. cm.

PROBLEMS.

1. The altitude of a cylinder is 6 in. and the radius of its base 2 in. Find its lateral area. Its total area. Its volume.
2. The volume of a cylinder is 2772 cu. cm. and its altitude 12 cm. Find its lateral area to one decimal place.
3. What must be the height of a tomato can to hold a quart (57.75 cu. in.) if its diameter is 4 in.?
4. The volume of an irregularly shaped object may be found by submerging it in a cylindrical vessel of water. When a stone is submerged in a cylindrical vessel 8 in. in diameter, the level of the water rises 4 in. Find the volume of the stone.
5. How many cubic yards of earth must be removed to dig a cistern 9 ft. in diameter and 10 ft. deep?
6. The cross-section of a tunnel is in the form of a square

with a side 16 ft., surmounted by a semicircle. The tunnel is 132 yards long. How many cubic yards of earth are removed in its construction?

7. An oil tank is 6 ft. in diameter and 15 ft. 3 in. long. How many gallons does it hold? (1 cu. ft. = 7.48 gallons, approx.)

8. What is the cost of cementing the side and bottom of a well 4 ft. 8 in. in diameter, and 30 ft. deep, at 45 cents per sq. ft.?

9. A boiler has 200 tubes 10 ft. long and 3 in. inside diameter, conveying the heat through the water. What is the area of the inside tube-heating surface?

10. A zinc disc, 8 in. in diameter, with a hole 1 in. in diameter in it, weighs 8 oz. Find its thickness. Use .26 lb. for the weight of 1 cu. in.

11. A cylinder 4 ft. 6 in. long and 1 ft. 4 in. in diameter is turned up in a lathe. The surface moves at the rate of 15 ft. a minute. The edge of the cutting tool is $\frac{7}{16}$ in. wide. How long will it take to turn up the whole cylinder?

12. A wooden cylinder is turned up on a lathe from a block 6 in. square. What part of the wood is cut away?

13. A pyramid with the base 15 cm. square is trimmed down to the largest possible cone. How much is the volume reduced?

14. A hollow cast-iron cylinder is to weigh just 100 pounds. It is to be 18 inches long with an inside diameter of 4 inches. How thick will it be? A cubic inch of cast-iron weighs .26 lb.

15. A brass tube whose inner diameter is 2 inches and outer diameter 3 inches is run from 50 lb. of brass. How long is it? A cubic inch of brass weighs .303 lb.

16. The piston of a pump is 8 in. in diameter, and makes a stroke of 20 in. How many gallons of water will it deliver in an hour, if it makes 30 strokes a minute? (231 cu. in. = 1 gal.)

17. By the method of § 157, the lateral area of a frustum of a right circular cone is seen to equal the *product of the slant height by half of the sum of the circumferences of the bases*. Find the lateral area of a frustum whose slant height is 3 ft., and the radii of whose upper and lower bases are 2 ft. and 4 ft., respectively.

18. How much galvanized iron is required to make a funnel with the radii of top and bottom 6 in. and 1 in., respectively, and slant height 10 in.?

19. It is proved in geometry that the volume of a frustum of a cone is obtained by the same rule as the volume of a frustum of a pyramid. (See § 154.) Find the volume of the frustum described in Ex. 17.

20. A bucket is 12 in. wide at the top and 8 in. wide at the bottom, and is 10 in. deep. How many gallons of water will it hold?

21. A round tank is 8 ft. wide at the top, 10 ft. at the bottom, and 10 ft. deep. How many gallons will it hold?

22. A contractor builds a cement cistern in the shape of a cylinder 10 ft. wide and 8 ft. high, surmounted by the frustum of a cone 18 in. in diameter at the top and 4 ft. high. How many gallons does it hold? How much cement is required, if the wall and floor are 8 in. thick?

23. A cubic foot of ivory weighs 114 lb. What is the weight of a billiard ball 2 in. in diameter?

24. A hollow spherical steel shell is 1 in. thick, and its outside diameter 10 in. Steel weighs 7.8 times as much as water, and a cu. ft. of water weighs about 1000 oz. Find the weight of the shell in pounds.

25. Two spheres of lead, of radii 2 in. and 4 in., respectively, are melted and recast into a solid cylinder of height 6 in. Show that the surface exposed is unchanged in amount.

26. Find the weight of a cast-iron spherical shell one inch thick and whose inside diameter is 7 inches, one cubic foot of cast-iron weighing 450 lb.

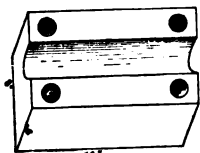
27. A cylindrical tank for supplying water to a locomotive is 24 ft. in diameter and 16 ft. high. Find its capacity in gallons. ($7\frac{1}{2}$ gallons = 1 cu. ft.)

28. A hollow sphere of brass is found to weigh 50 lb. its external diameter is 10 in. What is its inside diameter? Its thickness? (1 cu. in. of brass weighs .308 lb.)

29. Three oil cans are of the same height. Two of them are each 10 inches in diameter and the other 15 inches. Compare the capacity of the large one with that of the other two.

30. A drum head 21 inches in diameter is cut from a sheet of $\frac{3}{8}$ -inch steel plate 22 inches square. What is the weight of the steel cut off? ($\frac{3}{8}$ -inch steel plate weighs 15.3 lb. per square foot.)

31. A 38-ft tank car has a tank 93 inches inside diameter and 34 feet long. How many gallons will it contain? (231 cu. in = 1 gal.)



32. Find the weight of a block of steel 4 inches by 8 inches by 14 inches, from which a half-round 3-inch groove is cut. It also contains 4 $1\frac{1}{2}$ -inch holes as shown in the figure. (A cubic inch of steel weighs .283 lb.)

CHAPTER V.

SOME APPLICATIONS OF RATIO, PROPORTION, AND VARIATION.

161. Ratio, proportion, and variation are very closely related subjects. Many of the problems of business really involve proportion. In fact, years ago proportion (often called the "Rule of Three") was used in solving most of the applied problems of arithmetic. In this chapter, however, ratio, proportion and variation will be applied only to certain kinds of problems in practical geometry and physical science.

162. Ratio. When one number is divided by another number of the same kind, the quotient is called the **ratio** of the two numbers.

For example, the ratio of 12 ft. to 4 ft. is $12 \text{ ft.} \div 4 \text{ ft.}$, or 3. Similarly, the ratio of 5 lb. to 7 lb. is $5 \text{ lb.} \div 7 \text{ lb.}$, or $\frac{5}{7}$.

A ratio is always an abstract number.

The ratio of a to b may be expressed by any of the notations used to express a quotient. Thus, the ratio of a to b may be written either $\frac{a}{b}$, $a:b$, $a \div b$, or a/b . Of these, the first two are most commonly used.

EXERCISES.

Find the ratios of the following :

1. 64 to 16.

4. 7 in. to 14 in.

2. 9 to 12.

5. 6 yd. to 2 yd.

3. $\frac{3}{8}$ to $\frac{5}{8}$.

6. 400 cm. to 8 cm.

7. 16 oz. to 22 oz. 11. 125 sq. m. to 5 sq. m.
 8. 3 mi. to 450 ft. 12. 160 sq. rd. to 4 acres.
 9. \$4.50 to \$30. 13. \$3.25 to \$4.20.
 10. 12 cu. ft. to 16 cu. ft. 14. $3\frac{1}{4}$ lb. to 12 oz.
 15. The circumference of a circle to the diameter; to the radius.
 16. The volume of a prism to that of a pyramid with equal base and altitude.
 17. The diagonal of a square to one side.
 18. The area of a circle with diameter 12 in. to that of a square with side 12 in.

163. Application of ratio to specific gravity. The *specific gravity* of a substance is the ratio of the weight of that substance to the weight of an equal volume of some substance taken as a standard. For the specific gravity of solids and liquids, distilled water is usually taken as the standard substance. For gases, either air or hydrogen is taken as the standard.

Thus, the ratio of the weight of a cubic inch of lead to the weight of a cubic inch of distilled water is 11.3. That is, the *specific gravity* of lead is 11.3. Likewise, the ratio of the weight of a cu. yd. of oxygen to the weight of a cu. yd. of air is 1.11. That is, the *specific gravity* of oxygen, referred to air, is 1.11.

The following specific gravities of substances have been found by experiment :

SPECIFIC GRAVITIES, REFERRED TO WATER.

Platinum	22.	Brass	8.4.	Sulphuric Acid	1.84.
Gold	19.3.	Steel	7.8.	Sea-water	1.025.
Mercury	13.6.	Tin	7.3.	Linseed oil	0.94.
Lead	11.3.	Zinc	7.2.	Ice	0.92.
Silver	10.5.	Cast-iron	7.	Alcohol	0.79.
Bronze	8.9.	Granite	2.7.	Petroleum	0.7.
Nickel	8.9.	Glass	2.5.	Oak	0.6.
Copper	8.9.	Sulphur	2.	Cork	0.24.

SPECIFIC GRAVITIES, REFERRED TO AIR.

Chlorine gas 2.24. Oxygen 1.11. Coal gas 0.42. Hydrogen 0.07.

The primary unit of weight in the metric system is the **gram**. It is the weight of 1 cu. cm. of distilled water at maximum density. It is equivalent to 0.035 ounce. A cu. dm. (also called a *liter*) of distilled water weighs 1000 gm. or a **kilogram**. (Why?) A cu. m. of distilled water weighs 1000 Kgm., called a **metric ton**.

NOTE.—Instead of the term *specific gravity*, the scientist uses the term **density**. Density is the weight of a unit volume of a substance. The unit of volume, in science, is the *cubic centimeter*, and the unit of weight, the *gram*. Hence the number of grams of density of a substance is the number that expresses the specific gravity of that substance.

The following weights are given for convenient reference :

WEIGHTS OF AIR AND WATER.

1 cu. cm. of air weighs 0.001293 gram. 1 cu. cm. of water weighs 1 gram. 1 cu. ft. of water weighs about 62.5 lb., or about 1000 oz.

EXAMPLE. Find the weight of 3 cu. ft. of steel.

1 cu. ft. of water weighs 62.5 lb.

Hence, 1 cu. ft. of steel weighs 7.8×62.5 lb.

Hence, 3 cu. ft. of steel weigh $3 \times 7.8 \times 62.5$ lb., or 1462.5 lb.

PROBLEMS.

1. What is the weight of a cubic foot of lead ? Of cast-iron ?
2. What is the weight of 5 cu. ft. of granite ? Of 10 cu. ft. of oak ?
3. What is the weight of 1 cu. ft. of copper ? Of glass ? Of bronze ?
4. What is the weight of 1 cu. in. of gold ? Of silver ? Of platinum ?
5. What is the weight of 3 cu. in. of mercury ? Of 12 cu. in. of nickel ?

6. A piece of zinc weighs 1125 lb. How many cu. ft. are there?

7. How many cu. ft. of glass weigh 1572.5 lb.?

8. Find the weight of 1 cu. yd. of cork. Of sulphur.

9. Find the weight of 10 gal. of sea-water.

10. Find the weight of 1 gal. of alcohol. Of 1 gal. of petroleum.

11. What is the weight of a block of ice 12 in. thick, 18 in. wide, and 36 in. long?

12. How many tons of ice will an ice-house hold that is 24' by 30' by 20', allowing $2\frac{1}{2}$ ft. on all sides and above and below for sawdust?

13. What is the weight of 2000 cu. cm. of oxygen? Of 175 cu. cm. of chlorine gas? Of 500 cu. cm. of coal gas? Of 1000 cu. cm. of hydrogen?

14. What is the specific gravity of chlorine gas, referred to hydrogen as a standard? Of oxygen? Of coal gas? Of air?

15. The specific gravity of good milk is 1.032. If 520 cu. cm. of the milk taken from a can in a certain dairy wagon are found by the inspector to weigh 532 grams, is the milk good?

16. If 40 cu. cm. of sodium are found to weigh 38.87 grams, find the density of sodium.

17. One cubic foot of a certain kind of limestone is found to weigh 187.5 lb. What is the specific gravity?

18. How many ounces of alcohol will a "two-ounce" bottle hold?

19. How many grams of mercury in a glass tube whose cross-sectional area is 1 sq. cm. and length 20 cm.?

20. A barometer consists of a glass tube sealed at the upper end and held in an upright position with the open end under the surface of mercury in a dish. The air is first removed from the tube, leaving a vacuum above the mercury. The weight or pressure of the atmosphere upon the surface of the mercury in the dish forces the mercury up into the tube. By measuring the height to which the mercury rises the atmospheric pressure is determined. What is the atmospheric pressure in grams per sq. cm. when the mercury stands 72 cm. high? How many pounds is this per sq. in.?



21. To what depth will a rectangular oak beam a foot square and 10 feet long sink in water? (It will sink until it displaces an amount of water whose weight equals the weight of the whole beam.)

22. How many yards of copper wire $\frac{1}{16}$ inch in diameter in a roll that weighs 260 lb.?

23. Find the weight per foot of a lead pipe of $1\frac{1}{2}$ in. bore and $\frac{3}{16}$ in. thick.

24. Find the weight of a cast-iron pipe 1.3 ft. long, of 10-in. radius, and $\frac{7}{8}$ in. thick.

25. What is the weight of a hollow steel pillar 10 ft. long, whose external diameter is 5 in., and internal diameter 4 in.? What is the diameter of a solid pillar of the same weight and length?

26. A hollow sphere of brass is found to weigh 50 lb. Its external diameter is 10 in. What is its internal diameter?

27. A grindstone when new is 6 ft. in diameter and has a 14 in. face. What has it lost in weight when it is worn down 2 inches? (The specific gravity of sandstone is 2.42.)

28. What weight of brass is cut off in turning a piece 1 in. in diameter and 1 ft. long from a bar $1\frac{1}{4}$ in. square and 1 ft. long?

29. A tank car has a tank 7 ft. 9 in. inside diameter and 34 ft. long. What weight of petroleum will it carry?

30. A tank 7 in. deep, 19 in. wide, and 1200 feet long (inside measures) is made of $\frac{3}{8}$ in. steel plate. Find its weight, no allowance being made for laps, joints, rivets or stiffening. (The tank is open the entire length at the top to allow the tenderscoop to take water while the train is in motion.)

164. Application of ratio to expansion of materials. Most substances, when heated, expand. Consequently, the amount of rise or fall of temperature may be measured by measuring the amount of expansion or contraction of a substance, such as mercury.



This principle is applied in the use of the mercurial thermometer, which is a sealed glass tube containing mercury and with a scale marked on it. There are two kinds of thermometer scales in common use: the **Fahrenheit**, and the **Centigrade**. In the Centigrade scale, the freezing point of water is marked 0° , and the boiling point 100° ; in the Fahrenheit scale, the freezing point of water is marked 32° , and the boiling point 212° . Hence, 180°F are equal to 100°C .

Degrees above 0 on each scale are usually indicated by the plus sign; below, by the minus sign. Thus, 70° above zero, Fahrenheit, is written $+70^{\circ}\text{F}$; and 12° below zero, Centigrade, -12°C .

It is easily seen that readings on the Centigrade scale may be converted into readings on the Fahrenheit scale, and vice versa, by the formulæ:

$$F = \frac{9}{5} C + 32, \text{ and } C = \frac{5}{9} (F - 32).$$

For example, if $C = -10^{\circ}$, $F = -\frac{9}{5} \times 10 + 32^{\circ}$, or $+14^{\circ}$; if $F = +59^{\circ}$, $C = \frac{5}{9} (59 - 32^{\circ})$, or $+15^{\circ}$.

When a substance, such as a steel rod, is heated through any number of degrees of temperature, the amount by which

it increases in length depends upon its original length. The ratio of the increase in length of a substance to the initial length, for a rise in temperature of 1°C , is called the **coefficient of linear expansion** of the substance. The following coefficients of linear expansion of substances have been found by measurements:

COEFFICIENTS OF LINEAR EXPANSION.

Divide each of the numbers by 100,000 to get the true coefficient.

Aluminum	2.34.	Silver	1.94.	Slate	1.04.
Copper	1.79.	Tin	2.27.	Oak (across fibre)	5.4.
Gold	1.45.	Zinc	2.9.	Oak (along fibre)	0.5.
Iron	1.2.	Brass	1.87.	Glass	0.9.
Lead	2.95.	Steel	1.11.	Platinum	0.9.

The ratio of the increase in volume of a substance to the initial volume, for a rise in temperature of 1°C , is called the **coefficient of cubical expansion** of the substance. The coefficient of cubical expansion is three times the coefficient of linear expansion.

NOTE. If a unit cube, *i. e.*, a cube whose edge is 1 unit, of a substance whose coefficient of linear expansion is a , be heated 1° , its edge will be $1+a$ units. By algebra, $(1+a)^3=1+3a+3a^2+a^3$, the volume. Since the coefficient of linear expansion a is a very small fraction, $3a^2$ and a^3 are so very small that they may be neglected, and the increase in volume be considered $3a$. Hence, the *coefficient of cubical expansion* is taken to be three times the *coefficient of linear expansion*.

The values for some substances not obtainable from the above table are:

COEFFICIENTS OF CUBICAL EXPANSION.

Divide each of these numbers (which hold between 0°C and 100°C) by 1000.

Alcohol	1.26.	Olive oil	0.8.	Pure water	0.43.
Mercury	0.18.	Petroleum	1.04.	Sea-water.	0.5.

PROBLEMS.

1. Metal consisting of 3 parts tin, 5 lead, and 8 bismuth, fuses at 212°F . Metal consisting of 4 parts tin, 4 lead, and 1 bismuth, fuses at 263°F . Metal consisting of 3 parts tin, 2 lead, fuses at 275°F ; 1 part tin, 1 lead, fuses at 304°F ; 1 part tin, 2 lead, fuses at 361°F . Convert these fusing points into the Centigrade scale.

2. What is the difference in temperature between 64°F and 64°C ? Between 49.78°C and 89.6°F ?

3. Express on the Centigrade scale the following melting points: lead, $+630^{\circ}\text{F}$; mercury, -38.99°F . In the Fahrenheit scale: aluminum, 700°C ; gold, 1100°C ; steel, 1400°C .

4. Express on the Fahrenheit scale the following boiling points: alcohol, $+78^{\circ}\text{C}$; ether, $+35^{\circ}\text{C}$; mercury, $+357^{\circ}\text{C}$; sulphuric acid, 338°C ; tin, 1500°C ; turpentine, 160°C .

5. A copper wire 20 ft. long at 0°C has what length at 30°C ?

6. A sheet of aluminum 1.5 in. wide at 28°C is how wide at 42°C ?

7. A lead pipe 1.25 in. in diameter at 212°F will have what diameter at 80°F ?

8. A platinum wire is used to convey the electrical current through the glass wall in an incandescent light bulb. Why? Why not use copper?

9. A steel connecting-rod is 12 ft. long at 10°C . What is the increase of length at 80°C ?

10. An iron pump rod is 1000 ft. long. What is its change of length for a change of 18°F ?

11. The temperature in a certain region varies from -10°F in winter to $+110^{\circ}\text{F}$ in summer. If the steel rails of a railroad through the region are laid when the temperature is $+50^{\circ}$,

and the rails are 28 ft. long, how much space should be left between them at the joints to allow for expansion?

12. If the slates on a roof are 10 inches wide at 72°F , which is the temperature when laid, what space must be left between them to allow for a rise of 80°F ?

13. An oak board is 2 in. thick, 6 in. wide, and 12 ft. long, at 42°C . What are the dimensions at 65°C ?

14. The pendulum of a clock is of brass, and 10 in. long at 70°F . What is its length at 40°F ?

15. The girder of a steel bridge is not fastened at the end because of expansion due to change of temperature. If the bridge is 68 ft. long, how much does it contract with a fall of 45°C ?

16. If 5 cu. cm. of silver are heated through a temperature of 60°C , what will be the resulting volume? 5 cu. cm. of zinc? 5 cu. cm. of tin?

17. If a sphere of lead contains 425 cu. in. at 20°C , what is its volume at 100°C ?

165. Proportion. The expression of equality of two ratios is called a **proportion**.

Thus, $\frac{4}{5} = \frac{12}{15}$, $\frac{3 \text{ lb.}}{4 \text{ lb.}} = \frac{\$18}{\$24}$, and $\frac{30 \text{ mi.}}{42 \text{ mi.}} = \frac{5}{7}$, are *proportions*.

The proportion $a/b = c/d$ is often written also $a:b = c:d$. The form $a:b::c:d$ is not now used as much as formerly.

In the proportion $a:b = c:d$, a , b , c , d are called the **terms**, a and d being the **extremes**, and b and c the **means**.

166. Principles in proportion. By multiplying or dividing equals, it is easily shown that the following principles in proportion are true:

I. *The product of the extremes equals the product of the means.*

Thus in $\frac{2}{3} = \frac{4}{6}$, multiplying each ratio by 18, $2 \times 6 = 3 \times 4$; in $\frac{a}{b} = \frac{c}{d}$, $ad = bc$; in $x:5 = 24:15$, $15x = 120$.

II. *Either extreme equals the product of the means divided by the other extreme.*

Thus, in $\frac{4}{5} = \frac{8}{10}$, multiplying by 5, $4 = \frac{5 \times 8}{10}$; in $x:3 = 12:18$, $x = \frac{3 \times 12}{18}$, or 2.

III. *Either mean equals the product of the extremes divided by the other mean.*

Thus, in $\frac{2}{3} = \frac{4}{6}$, $4 \times 3 = 2 \times 6$, and hence dividing by 3, $4 = \frac{2 \times 6}{3}$. In $\frac{7}{n} = \frac{21}{24}$, $n = \frac{7 \times 24}{21}$, or 8.

NOTE.—If the terms of a proportion are concrete numbers, so that the above principles imply multiplying by a concrete number, the difficulty is avoided by making each term of either ratio abstract. *E. g.*, $\frac{\$7}{\$12} = \frac{14 \text{ books}}{24 \text{ books}}$ is equivalent to $\frac{7}{12} = \frac{14}{24}$, each ratio being abstract.

167. If $a:b = c:d$, a and b are said to be *directly proportional* to c and d .

168. If $a:b = d:c$, a and b are said to be *inversely proportional* to c and d .

EXERCISES.

1. Find the value of x , if (a) $x:2 = 50:10$; (b) $x:21 = 5:4$; (c) $4:x = 12:3$.

2. Find the value of V , if (a) $V \text{ cu. cm.} : 7 \text{ cu. cm.} = 9:2$; (b) $6:2.5 = V:5$; (c) $2 \text{ lb.} : 15 \text{ lb.} = 3 \text{ cu. cm.} : V \text{ cu. cm.}$

3. Find the value of t , if (a) $6 \text{ sec.} : t \text{ sec.} = 20 \text{ cm.} : 4 \text{ cm.}$; (b) $100 \text{ ft.} : 24 \text{ ft.} = t \text{ sec.} : 2 \text{ sec.}$; (c) $.05:2 = 4.25:t$.

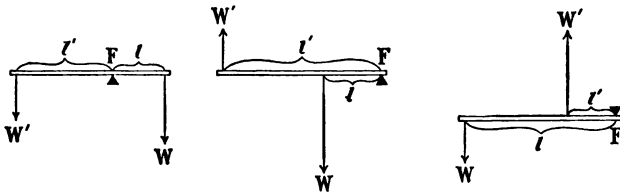
4. Find the value of a , if (a) $8:a = a:2$; (b) $3:a = a:12$.

5. Are 2 and 6 *directly* proportional to 5 and 15? Are 20 and 4 directly proportional to 5 and 2? Why?

6. Are 12 and 8 *inversely* proportional to 24 and 36? Are 4 and 96 inversely proportional to 164 and 7?

169. Applications of proportion to simple machines. In every simple machine there are two forces involved: the **resistance**, or force to be overcome, and the **effort**, or force necessary to overcome the resistance. The relation between the resistance and effort depends upon the nature of the machine and upon the dimensions of its parts.

I. The lever. In the *lever*, the resistance W and the effort W' are applied at different points of a rigid bar which revolves freely about a point of support called the **fulcrum**. There are three classes of the lever as shown in the figures. The dis-



tances l and l' , from the fulcrum to the points of the lever where the resistance and effort, respectively, are applied are called the *arms* of the lever. It is shown that

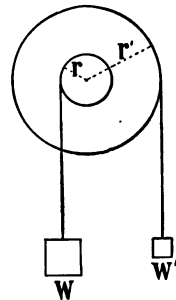
The resistance and effort are inversely proportional to their distances from the fulcrum; that is, $W : W' = l' : l$.

The beam balance, common steel-yard, scissors, pincers, crow-bar, etc., are familiar examples of levers.

II. Wheel and axle. This consists of a wheel and cylinder constituting a rigid body, and revolving about the same axis. The effort W' is applied to the circumference of the wheel, and the resistance W to that of the cylinder by means of a cable winding around it in a direction opposite to the motion of the wheel. It is shown that

The resistance and effort are inversely proportional to the radii of the cylinder and wheel respectively; that is, $W : W' = r' : r$.

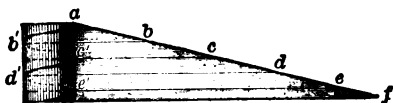
The windlass and capstan are familiar examples of the wheel and axle.



Modifications of the wheel and axle are wheels on separate axes, the motion being transmitted by means of belts, or by means of teeth in the circumferences of the wheels.

III. The inclined plane. The inclined plane is a surface inclined at an angle to a horizontal surface. The weight W of an object on an inclined plane tends to cause the object to move down the incline. The force, or effort W' , exerted up the incline, and sufficient to hold the object in position, depends upon the relation of the length l to the height h of the incline. It is shown that

The weight and effort are directly proportional to the length and height of the incline; that is, $W:W' = l:h$.



IV. The screw. The screw is a particular form of the inclined plane. This can be seen by winding a triangular piece of paper about a cylinder as shown in the figure. The distance between two consecutive threads is called the *pitch* of the screw. The effort

is applied at the end of a lever. The relation between the resistance W and the effort W' is the same as for the inclined plane. If the pitch is d , and the lever arm l , it is shown that

The weight and effort are directly proportional to the distance traversed by the effort in one revolution and the pitch of the screw; that is, $W:W' = 2\pi l:d$.

The screw is used for overcoming great resistances, such as raising buildings, propelling ships, in the letter-press, etc.

EXERCISES.

1. What effort is required to lift a weight of 800 lb. by means of a lever of which the weight-arm is 6 in. and the effort-arm 2 ft. 6 in.?

2. A safety-valve of an engine consists of a circular hole, $\frac{1}{4}$ in. in diameter, closed by a plunger attached to a light horizontal hinged bar 1 in. from the hinge. A weight of 1 lb. slides on the bar. How far from the hinge must it be set if the steam is to blow off at 100 lb. on the square inch?

3. A pump handle is 3 ft. 8 in. long, and works on a pivot 4 in. from the end attached to the pump rod. What force is applied to the pump rod when the handle is pushed down with a force of 10 lb. weight?

4. An oarsman, in rowing, uses an oar with which the distance from the rowlock to the water is 3 times the distance from the hand to the rowlock. What is the propelling force, if he pulls with a force of 48 lb.?

5. A man lifts a 1000-pound stone by means of a crow-bar. The point of the crow-bar is placed under the edge of the stone, and a small stone for fulcrum is placed 10 in. back from it. The crow-bar is grasped at a distance of 6 ft. 8 in. from the fulcrum. What force is exerted in lifting the stone?

6. Two children play "see-saw." One weighing 80 lb. sits 6 ft. from the point of support of the board, and the other 5 ft. Find the weight of the second child.

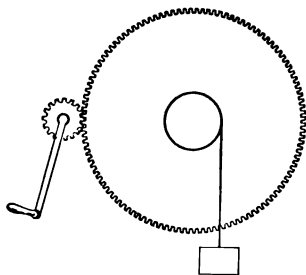
7. The length of the handles of a wheel-barrow is 5 ft., and the center of gravity of a load of 275 lb. is 18 in. from the hub of the wheel. What force is required to lift it?

8. The crank to the windlass of a well is 18 in. long, and the cylinder upon which the rope is wound, 9 in. in diameter. What force is necessary to lift from the well a bucket of water weighing 48 lb.?



9. Two men working at a capstan walk in a circle 8 ft. in diameter, and each exerts a force of 60 lb. With each turn of the capstan 2 ft. of rope are pulled in. What pull is exerted along the rope?

10. A horse walking in a circle 15 ft. in diameter moves a house by means of a capstan 18 in. in diameter. The horse exerts a pull of 1200 lb. What is the resistance of the house?



11. A stone weighing 756 lb. is lifted into place in a wall by means of a cable wound round an axle of 5 in. radius. To the axle is rigidly attached a toothed wheel 20 in. in radius which is driven by another toothed wheel 4 in. in radius. The latter wheel is turned by means of a crank 18 in. long.

How much force must be applied to the crank?

12. A shaft has upon it two pulleys. The small one is 8 inches in diameter and drives machinery with a resistance of 196 lb. The shaft is turned by the other pulley, which is 22 inches in diameter. What effort must be applied to the large pulley?

13. If the wheels of an electric car are 2 ft., the axle cog-wheel 8 in., and the cog-wheel attached to the motor 12 in. in diameter, and if the resistance of the car is 1260 lb., what force must the motor supply to move the car?

14. There are four pulleys on a shaft driven by a fifth pulley. The driving pulley is 36 in. in diameter, and the others are 12 in., 10 in., 8 in., and 6 in. in diameter, respectively. The latter drive machinery with resistances of 50 lb.,

36 lb., 48 lb., and 30 lb., respectively. What force must be communicated to the large pulley?

15. The current of water strikes an over-shot water wheel with a force of 500 lb. On the shaft of this wheel, which is 8 ft. in diameter, is a pulley 2 ft. in diameter. The latter is connected by a belt to another pulley which turns a long shaft bearing many pulleys in a mill. How much force is communicated to this shaft?

16. The drive wheel of a certain lawn-mower is 10 in. in diameter, and contains 64 teeth. The little ratchet wheel, which propels the blades and which is driven by the first wheel, contains 10 teeth. The blades describe circles 6 in. in diameter. By how much is the effort applied to the drive wheel reduced at the blades?

17. The drive-wheel of a locomotive 5 ft. in diameter has the connecting rod attached to a pivot 8 in. from the axle. What is the maximum force that tends to move the engine forward, if the force along the connecting rod due to the steam is 25000 lb.?

18. If a barrel weighs 200 lb., what force is required to roll it up an incline 8 ft. long into a wagon 3 ft. high? If it weighs 240 lb.?

19. If a stone weighs 850 lb., what force is required to pull it on rollers up an incline 12 ft. long, and place it upon a wagon 30 in. high?

20. What force is required to pull a loaded wagon weighing 3800 lb. up a slope 30 ft. high and 105 ft. long, neglecting friction?

21. A car weighing 15 tons stands on an inclined track 10 times as long as it is high. With what force does it tend to run down the track?

22. Ice is pulled up an incline 50 ft. long and 20 ft. high,

into an ice-house. What force, neglecting friction, is necessary to pull up a block weighing 200 lb.?

23. Shavings from a spoke mill are carried up an incline 40 ft. long to a height of 10 ft. by means of a belt with slats on it, and thrown into wagons. If the shavings weigh 12 lb. to 10 ft. of the carrier, what force due to the shavings alone is required to run the carrier?

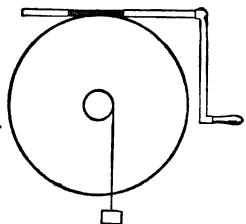
24. The wheel of a letter-press is 14 in. in diameter, and the screw advances $\frac{1}{4}$ in. at each turn of the wheel. What pressure will a force of 15 lb. at the wheel exert upon the articles in the press?

25. A jack-screw whose pitch is $\frac{3}{8}$ in. is operated by a lever 24 in. long. What force is required to lift the corner of a house with it, if the weight is 3 tons?

26. If the pitch of a jack-screw is $\frac{3}{4}$ in., and the lever arm 30 in., what weight may be lifted by it with a force of 100 lb.?

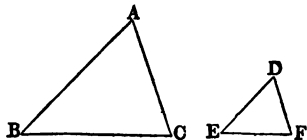
27. Two iron bars are bolted together by a bolt of which the distance between the threads is $\frac{1}{8}$ inch. If the tap is tightened by means of a wrench of which the handle is 14 inches long, with a force of 25 lb., what is the strain on the threads? Neglect friction.

28. A carpenter wishes to make a bench-vise having a screw with a pitch of $\frac{2}{3}$ inch, and a lever of such length that a force of 10 lb. applied at the lever will cause a pressure of 2200 lb. on a block in the vise. How long must the lever be?



29. The wheel of an endless screw which has 4 teeth to the inch, is 24 inches in diameter and the axle 4 inches. The crank of the screw is 15 inches long. What resistance will the machine overcome, if a force of 40 lb. is applied at the crank? What resistance will the same machine overcome if the crank of the screw is 20 in. and the effort 75 lb.?

170. Application of proportion to similar figures. Two figures, such as two squares, two triangles whose angles are respectively equal, two circles, two cubes, two spheres, etc., which have the same shape are called **similar**. In geometry it is shown that in two similar figures :



I. *Two corresponding lines are proportional to any other two corresponding lines ;*

II. *The areas are proportional to the squares on any two corresponding lines ;*

III. *The volumes are proportional to the cubes on any two corresponding lines.*

EXAMPLE. The base and altitude of a triangle are 8 in. and 6 in., respectively, and the base of a similar triangle is 12 in. Find its altitude.

We have $x \text{ in.} : 6 \text{ in.} = 12 \text{ in.} : 8 \text{ in.}$

Hence, $x \text{ in.} = \frac{6 \times 12}{8} \text{ in.}$, or 9 in.

Proportion is involved in the *drawing to a scale* of maps, plans for buildings, machine designs, etc. The map or plan of a thing is similar to the thing represented. Thus, the distances between points on a map of the United States are proportional to the actual distances on the earth's surface which they represent.

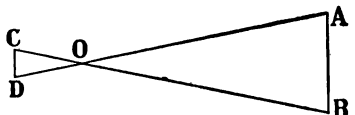
PROBLEMS.

1. Two corresponding sides of two similar triangles are 4 ft. and 12 ft., respectively. The other two sides of the first triangle are 5 ft. and 6 ft. Find the corresponding sides of the second triangle.

2. Two corresponding sides of two similar triangles are 18 yd. and 26 yd., respectively, and the area of the first is 104 sq. yd. Find the area of the second.

3. The diameters of two spherical steel balls are 6 in. and 8 in., respectively. If the smaller weighs approximately 31.87 lb., what is the weight of the larger?

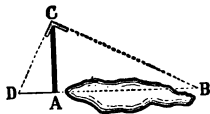
4. Of two similar iron castings, the smaller weighs 128 lb. Their lengths are 20 in. and 32 in., respectively. Find the weight of the larger.



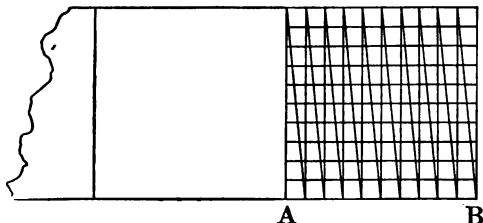
5. Sailors and others use the following method of estimating the distance DA to an object A. With the left eye closed, the finger is pointed, at arm's length, towards A. Then the right eye is closed and the left eye opened, when the object appears to have moved through the distance AB. The distance AB, being at right angles to the line of sight, is estimated. The distance CD between the eyes of the average person is 3.1 in., and the distance DO to the end of the finger 31 in. If AB is 500 ft., what is the distance to A? If AB is apparently 12 ft.? Estimate in this way the distances to objects about you.

6. A rod 6 ft. high casts a shadow 4 ft. 6 in. long, and at the same time a tree casts a shadow 51 ft. long. How high is the tree?

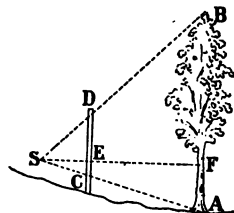
7. In the 16th Century the distance from A to the inaccessible point B was determined by erecting a vertical staff AC, and placing upon this an instrument resembling a carpenter's square, directing one arm towards B, and noting on the ground the point D toward which the other arm pointed. By measuring AC and DA, and using similar triangles, AB was estimated. If AC=5 ft. and DA=6 in., what is AB? If AC=6 ft. and DA=4 in., what is AB?



8. In the annexed figure of a "diagonal scale," which is used in measuring or laying off very short distances, AB is an inch. Show how, by means of this scale and a pair of compasses, to lay off 0.1 in.; 0.3 in.; 0.5 in.; 0.15 in.; 0.37 in.; 1.23 in.



9. In forestry, when shadows cannot be used, the height AB of a tree is found as follows: A staff is held in an upright position CD. A man at S sights across the staff to the foot and to the top of the tree. An assistant notes the points C and D where the lines of vision cross the staff, and measures CD. The distance SE and SF are measured. If $CD = 4$ ft., $SE = 3$ ft., and $SF = 49$ ft. 6 in., find AB. If $CD = 4$ ft. 2 in., $SE = 2$ ft. 8 in., and $SF = 37$ ft. 5 in., find AB.



10. Two similar cylindrical tanks have diameters 4 in. and 3 ft., respectively. The smaller contains 72 cu. in. Find the volume of the larger.

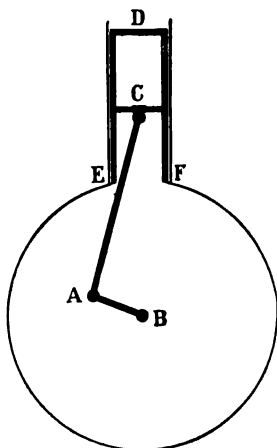
11. The areas of two similar rectangles are 165 sq. in. and $371\frac{1}{4}$ sq. in., respectively. The sides of the smaller are 11 in. and 15 in., respectively. Find the sides of the larger.

12. If, in a map, the distance between two cities 250 miles apart is represented by a distance of 2.5 in., what should represent the distance between two places which are 372 miles apart?

13. The room of a house is 16 ft. by 18 ft. If, in the plan, the width is represented by a line .8 in. long, what should represent the length?

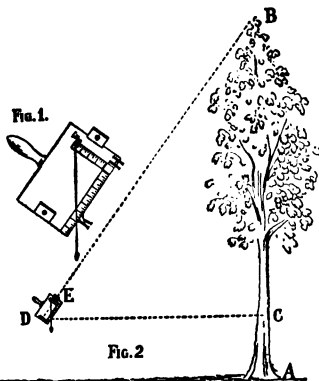
14. The dimensions of a rectangular piece of land are 220 yd. by 300 yd. If, in a plot of it, the width is made $1\frac{1}{4}$ in., what will be the length?

15. The surfaces of two similar bodies are 256 sq. cm. and 824 sq. cm., respectively. The volume of the first is 64 cu. cm. Find the volume of the second.



16. In constructing a gas engine, the piston D, which is in the form of an inverted cup, is 5 in. in diameter; the crank AB is 5 in. between the centers of the pivots; and the connecting rod AC is 17 in. between the centers of the pivots. How far from the mouth of the cup must the pin C be adjusted in order that the connecting rod may just clear the edge of the cup at E and F at each stroke, the diameter of the rod AC being 1 in.?

17. In Faustman's "height measure" (See fig. 1), used by foresters in finding the heights of trees, the edge of the frame HK and the sliding rule FH are marked off to a scale. FK is a plumb line.



In the similar triangles FHK and DCB, FH corresponds to DC. When DC is 50 ft., and FH set at 5 in., HK is 7 in. Find BC. When, with another tree, DC=64 ft. and FH=6 in., HK=10. Find BC. When DC=42 ft. and FH= $3\frac{1}{2}$ in., HK=6 in. Find BC.

18. In mechanics, forces are

represented by lines drawn to a scale. If 48 lb. is represented by a line 4.8 in. long, what will represent 39 lb.? 125 lb.? If 168 lb. is represented by 8.4 in., what will represent 92 lb.? 256 lb.?

19. A steel shaft in a machine being designed is 3 in. in diameter and 6 ft. 10 in. long. In the design, the diameter is represented by a line $\frac{3}{16}$ in. long. By what must the length be represented?

171. Variation. A quantity whose value changes is called a **variable**. If two variable quantities have a constant ratio, one is said to *vary as* the other.

Thus, if the diameter of a circle increases, the circumference increases also, but the ratio of the circumference to the diameter is constant, and equals π . This is expressed by saying that the *circumference of a circle varies as the diameter*.

If one quantity varies as another, any two values of the first will be directly proportional to the corresponding values of the second.

E. g., the weight of a substance varies as its volume. If the volume is doubled, the weight is doubled also. Hence, if w and v , the weight and volume, respectively, of a substance, change to w' and v' , respectively, then $w:w' = v:v'$.

If two variable quantities have a constant product, one is said to *vary inversely as* the other. One increases as the other decreases.

Thus, the time required for a railway train to travel 200 mi. varies inversely as its speed; because, if t and r represent the time and rate, respectively, then $t \times r = 200$.

If one quantity varies inversely as another, any two values of the first will be inversely proportional to the corresponding values of the second.

E. g., if a varies inversely as b , and when a changes to a' , b changes to b' , then $a:a' = b':b$.

Certain simple applications of the subject of variation will be discussed in the following sections.

172. The application of variation to falling bodies. It is proved in mechanics that if a body is let fall from rest, its velocity increases from second to second, and that

I. The velocity at any time varies directly as the number of units of time (seconds) that have passed.

II. The distance passed through at any time, varies directly as the square of the number of units of time (seconds) that have passed.

It is found experimentally that a body starting from rest falls approximately 16 feet the first second, and that the velocity at the end of the first second is approximately 32 feet per second.

PROBLEMS.

1. If a body starting from rest falls 16 ft. the first second, how far will it fall in 2 seconds? In 3 seconds? In 4 seconds? In 5 seconds?

2. If a body starting from rest falls 16 ft. the first second, how far does it fall during the second second? The third second? The fourth second?

3. How far will a body starting from rest fall in 10 seconds? How far during the tenth second? How far in 1 minute?

4. If a falling body starts from rest, and has a velocity of 32 ft. per second at the end of the first second, what is its velocity at the end of the second second? At the end of the third second? At the end of the tenth second? At the end of a minute?

5. For how many seconds has a body fallen to have a velocity of 192 ft. per second?

6. How long does it require an object starting from rest to fall 144 ft. ? ($x^2 : 1^2 = 144 : 16$.)

7. A brick dislodged from the wall of a building strikes the ground in $2\frac{1}{2}$ sec. How high is the wall?

8. A stone thrown just over the top of a tree, strikes the ground in 6 seconds from the time that it starts upward. Find the approximate height of the tree.

NOTE.—The time going up equals the time coming down.

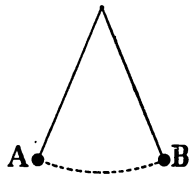
9. With what vertical velocity must a bullet have left a gun, if it fell back to its starting place in just 9 seconds after it left the gun? How high did it go? (See note Ex. 8.)

10. A mail bag is dropped from a train at a height of 4 ft. above the platform, while the train is going 30 mi. per hour. How far in advance of its starting point does the bag strike the ground?

11. An object dropped from a balloon strikes the ground in 6 seconds. At what velocity is the object falling when it strikes the ground, and how high is the balloon?

173. The application of variation to the vibration of the pendulum. The time of vibration of a pendulum is that required for it to swing from A to B. It depends upon the length of the pendulum, and the latitude of the place.

The time of vibration of a pendulum at any given place varies directly as the square root of the length.



Thus, at New York a pendulum 4 ft. long requires 1.1 sec. for a vibration; one 1 ft. long requires 0.55 sec. for a vibration.

PROBLEMS.

1. The length of a seconds pendulum (making a vibration in

1 sec.) in the latitude of New York is 39.1 in. What time is required for the vibration of a pendulum 9.78 in. long?

2. How long is a pendulum which vibrates three times in a second?

3. What is the length of a pendulum which vibrates 4 times in 3 seconds?

4. A pendulum 94 ft. long is made by suspending a cannon ball by means of a wire from the dome of a large building. What is the time of its vibration?

5. A clock whose pendulum beats seconds, gains 2 minutes a day. How much too short is the pendulum?

6. A clock whose pendulum vibrates in a half second, is found to gain 30 seconds a day in winter. How much has the pendulum contracted?

7. A clock whose pendulum is 20 in. long loses 10 min. a day. How much must the pendulum be shortened?

174. The application of variation to Boyle's Law. The volume of a gas depends upon the amount of pressure upon it, the greater the pressure the less the volume. It was shown by Robert Boyle, an English physicist in the seventeenth century, that, if subjected to pressure:

The volume of a gas varies inversely as the pressure upon it.

If V is the volume of a gas under the pressure P , and the volume changes to V' when the pressure changes to P' , then $V : V' = P' : P$.

PROBLEMS.

1. A room has a capacity of 150 cu. yd. The barometric pressure rises from 28 in. to 30 in. How many cu. yd. of air in the room at the higher pressure have entered during the rise?

2. Twenty-four cu. in. of air under a pressure of 40 lb. will have what volume when the pressure is increased to 100 lb.?

3. When the atmospheric pressure shown by the barometer is 76 cm., 1000 cu. cm. of air weigh 1.293 grams. What is the weight when the atmospheric pressure is 80 cm.?

4. If a toy balloon contains 2.5 quarts of gas when under a pressure of 1 atmosphere, to what size will it shrink if subjected to a pressure of 5 atmospheres?

5. The pressure of the atmosphere on a square inch of surface is approximately 15 lb. The bore of a popgun is $\frac{1}{16}$ sq. in. in cross-sectional area. The air in the popgun is compressed to $\frac{1}{12}$ of its original volume before the pellet is discharged. With what force is the pellet discharged?

6. Air in a cylinder at atmospheric pressure is compressed to $\frac{1}{10}$ of its volume. What pressure does it exert per square inch on the walls of the cylinder?

175. Intensity of light. The amount of illumination, or intensity of light thrown upon a surface from a given source depends upon the distance of the source. It is shown that:

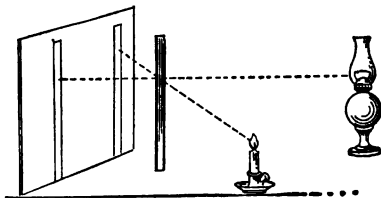
The intensity of the illumination varies inversely as the square of the distance from the source of light.

Thus, a standard candle placed 2 ft. from an open book will illuminate it with 4 times the intensity that it will when placed 4 ft. away. That is, $1:\frac{1}{4} = 4^2:2^2$.

This principle is used in determining the "candle power" of light.

PROBLEMS.

1. *The candle power of a lamp is the ratio of its illuminating power to that of a standard candle.* A kerosene lamp and a standard candle are placed so that they cause a rod to cast shadows upon a surface or screen. The shadows are of equal in-



tensity when the candle is 18 in. and the lamp 36 in. from the screen. What is the candle power of the lamp?

2. A kerosene lamp and a standard candle cast shadows of equal intensity upon a screen when the candle is 15 in. and the lamp 48 in. from the screen. Find the candle power of the lamp.

3. If a standard candle is placed 20 in. from the screen, how far from the screen will a lamp have to be placed, if its candle power is 9, in order to illuminate the screen with equal intensity?

4. A standard candle 1 ft., and an incandescent electric light 4 ft. from a screen, illuminate it with equal intensities. What is the candle power of the incandescent light?

5. A 16-candle power incandescent light at 25 in., and an oil lamp at 8 in. from a screen, illuminate it with equal intensities. What is the candle power of the lamp?

6. A cluster of twelve 16-candle power incandescent lights 10 ft., and an arc light 25 ft. from a wall, illuminate it with equal intensities. Find the candle power of the arc light.

7. How many incandescent lamps will be required to produce the same intensity of illumination at 6 ft. distance that is produced by 1 such lamp at 10 in. distance?

176. Electrical currents. A wire offers *resistance* to a current of electricity which it is conducting. The resistance is expressed as a number of **ohms**. An *ohm* is the resistance offered by a column of mercury 1 sq. mm. in cross-sectional area, 106.3 cm. long, and at 0°C.

The *quantity* or *strength* of a current of electricity flowing through a wire is measured in **amperes**. An *ampere* is the quantity of current that will deposit 4.025 grams of silver an hour in passing through a certain solution of nitrate of silver

The *pressure* required to force a current of electricity through wire is called **electro-motive-force** (*e. m. f.*). It is measured in **volts**. A *volt* is the pressure required to force 1 ampere of current through 1 ohm of resistance.

It is shown in physics that:

The resistance of a wire varies directly as its length and inversely as its cross-sectional area, or inversely as the square of the diameter.

E. g., if the resistance of 352 ft. of wire of a certain diameter is .09 ohm, the resistance of 1 mi. of it is 1.35 ohms. ($x : .09 = 5280 : 352$.)

Likewise, if the resistance of 1 mi. of copper wire 1.4 mm. in diameter is 8.29 ohms, the resistance of 1 mi. of copper wire 1.14 mm. in diameter is 12.50 ohms. ($x : 8.29 = 1.4^2 : 1.14^2$.)

Many problems in electricity may be solved without stating them in the form of proportion. Thus, either the resistance, the electro-motive-force, or the strength of current in a circuit can be obtained, if the other two are known, by the principle in electricity that:

The strength of current equals the electro-motive-force divided by the resistance; i. e.,

$$C = \frac{E}{R}$$

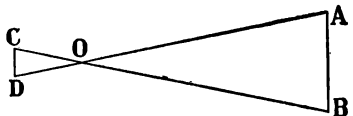
where C=number of *amperes*, E=number of *volts*, and R=number of *ohms*.

PROBLEMS.

1. What must be the diameter of a wire 40 ft. long to offer the same resistance to an electrical current as a wire 10 ft. long and 1 mm. in diameter? One-fourth as much resistance?
2. What length of wire 2 mm. in diameter will have the same resistance as 5 m. of wire 4 mm. in diameter?
3. If the resistance of a wire 10 m. long and 1 mm. in

3. The diameters of two spherical steel balls are 6 in. and 8 in., respectively. If the smaller weighs approximately 31.87 lb., what is the weight of the larger?

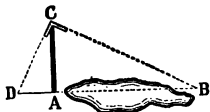
4. Of two similar iron castings, the smaller weighs 128 lb. Their lengths are 20 in. and 32 in., respectively. Find the weight of the larger.



5. Sailors and others use the following method of estimating the distance DA to an object A. With the left eye closed, the finger is pointed, at arm's length, towards A. Then the right eye is closed and the left eye opened, when the object appears to have moved through the distance AB. The distance AB, being at right angles to the line of sight, is estimated. The distance CD between the eyes of the average person is 3.1 in., and the distance DO to the end of the finger 31 in. If AB is 500 ft., what is the distance to A? If AB is apparently 12 ft.? Estimate in this way the distances to objects about you.

6. A rod 6 ft. high casts a shadow 4 ft. 6 in. long, and at the same time a tree casts a shadow 51 ft. long. How high is the tree?

7. In the 16th Century the distance from A to the inaccessible point B was determined by erecting a vertical staff AC, and placing upon this an instrument resembling a carpenter's square, directing one arm towards B, and noting on the ground the point D toward which the other arm pointed. By measuring AC and DA, and using similar triangles, AB was estimated. If AC=5 ft. and DA=6 in., what is AB? If AC=6 ft. and DA=4 in., what is AB?



14. What current will an *e. m. f.* of 250 volts send through a resistance of 12.5 ohms?

15. What *e. m. f.* is required to light an incandescent lamp of 80 ohms resistance, if it takes a current of 0.75 ampere?

16. A Daniell cell has an *e. m. f.* of 1.1 volts and an internal resistance of 4 ohms. What current will it send through an external resistance of 2 ohms?

17. A Bunsen cell has an *e. m. f.* of 1.8 volts and an internal resistance of 0.9 ohm. Through what resistance will it send 1.5 amperes?

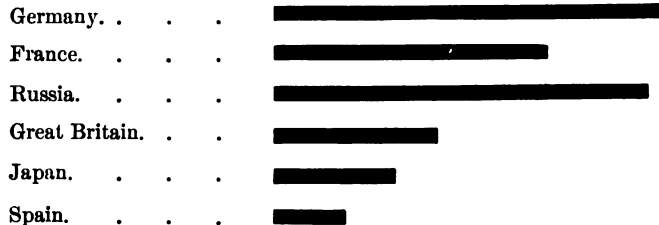
18. How many volts are required to produce a current of 40 amperes through a circuit in which there are 10 arc lamps, if the resistance of the dynamo is 4 ohms, that of each lamp 2.5 ohms, and that of the rest of the circuit 2 ohms?

CHAPTER VI.

GRAPHIC REPRESENTATION OF STATISTICS.

177. Quantities represented by lines. For purposes of showing the relations between quantities more clearly, such quantities are often represented by the lengths of lines, accurately drawn to a scale.

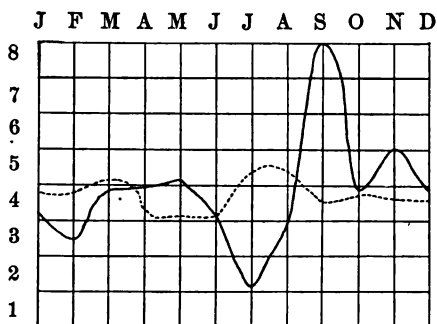
Thus, the estimated war strengths of the armies of some of the most powerful nations of Europe and Asia in 1907 were : Germany, 1,840,000 men ; France, 1,290,000 men ; Russia, 1,800,000 men ; Great Britain, 750,000 men ; Japan, 600,000 men ; Spain, 310,000 men. These may be represented by lines as follows, using a scale of 900,000 men to 1 inch :



178. Relations shown by curves. If there are two sets of quantities to be represented, such as time in months, and rainfall in inches, a curve may be drawn to show their relations. For this purpose, *squared paper* is used ; *i. e.*, paper ruled into equal small squares.

For example, the monthly precipitation, in inches, at New York for 1907 is given in the following table :

MONTH.	1907	MEAN.
Jan.	3.26	3.77
Feb.	2.52	3.74
Mar.	3.80	4.08
April	3.89	3.37
May	4.08	3.18
June	3.29	3.25
July	1.18	4.38
Aug.	2.48	4.53
Sep.	8.00	3.69
Oct.	3.82	3.70
Nov.	5.05	3.50
Dec.	3.91	3.39



The accompanying curve (continuous line) shows the variation of rainfall from month to month in 1907. Two lines at right angles are taken as base lines. Then the time in months is measured off to the right, one space to the month, and the corresponding rainfall is measured off upwards, one space to the inch. This gives a set of points. Thus, for April we count over to the fourth line ; and since the rainfall in April was 3.89 in., we count upwards on this line 3.89 spaces, locating a point. By joining the points for the various months we get the curve.

Similarly, the dotted line shows the normal variations for the last 37 years. We are able thus to see clearly the departure from the normal of the precipitation for the year.

EXERCISES.

1. The areas of the continents, in millions of square-miles, are : North America 8, South America 6.85, Europe 3.8, Asia 17, Africa 11.5, and Australia 3. Represent these areas by lines, using the scale of 6 millions of sq. mi. to the inch.

2. The distances of the planets from the Sun, in millions of miles, are as follows : Mercury 36, Venus 67, Earth 93, Mars 142, Jupiter 483, Saturn 886, Uranus 1782, Neptune 2792. Represent these distances by lines.

3. The following are the death rates, due to consumption,

per 10,000 population, by nationalities in an eastern city in 1906: American 7.5, English 29.2, German 24.8, Italian 30.1, Irish 52.5, Polish 9. Represent these by lines.

4. The number of paupers (percentage of population) in almshouses in the United States, was: in 1880, 0.132; in 1890, 0.1166; in 1903, 0.0876. Represent these statistics graphically by lines.

5. The value, in millions of dollars, of agricultural implements exported from the United States has been as follows: during 1880, 2.25; during 1885, 2.56; during 1890, 3.86; during 1895, 5.41; during 1900, 16.1; during 1905, 20.72. Show, by a curve, the growth of exportation of agricultural implements.

SUGGESTION: Mark off the years as the months were marked off in § 178. Then measure off the values of the exports on the corresponding year-lines. Join the points thus located by means of a smooth curve.

6. The population of the United States, in millions, for certain years has been as follows: in 1800, 5.31; in 1850, 23.19; in 1880, 50.16; in 1900, 76.30; in 1906, 84.15. Show the growth by means of a curve.

SUGGESTION. In marking off the year-lines, take proper account of the length of time between them.

7. The following average weights of men 5 ft. 10 in. in height at different ages are based upon the life insurance records:

Age:	15—24	25—29	30—34	35—39	40—44	45—49	50—54	55—59	60—64
Wt.:	154	159	164	167	170	171	172	173	174

Draw a curve representing the growth in weight.

8. The following are the numbers of children (in percentage of population) that have been attending public secondary schools in various years:

1889-90, .86; 1890-91, .85; 1891-92, .88; 1892-93, .89; 1893-94, .45; 1894-95, .53; 1895-96, .56; 1896-97, .59; 1897-98, .63; 1898-99, .66; 1899-1900, .70; 1900-01, .72; 1901-02, .72; 1902-03, .76; 1903-04, .80.

Draw a curve to show the increased attendance in public secondary schools.

9. Hastings measured (in kilograms) the strength of the right fore-arms of 5,476 children, of ages from 5 years to 16 years. He found the average strength as follows :

Age.....	5	6	7	8	9	10	11	12	13	14	15	16
Strength.....	4.9	7.0	9.2	10.6	13.1	14.7	18.0	19.7	22.6	25.4	28.9	33.3

Draw a curve to show the relation of the growth of strength to the increase of age.

10. The following table gives the death rates from tuberculosis per 10,000 population, in four cities of New York of similar size, for a period of 10 years beginning in 1898. Draw on the same chart the curves showing the variations for each city, and thus compare their death rates.

Year.	1898	1899	1900	1901	1902	1903	1904	1905	1906	1907
Auburn.....	18.8	13.5	15.2	13.3	9.2	18.0	15.2	11.9	13.1	14.2
Geneva.	14.0	13.4	10.5	11.5	10.5	13.4	16.3	11.4	10.6	12.7
Oswego.....	13.1	14.4	18.9	15.8	16.2	10.3	14.9	17.6	17.2	8.8
Canandaigua..	8.5	1.6	11.3	9.7	8.1	14.6	16.3	15.0	12.2	5.5

11. The percentages of all deaths in the city of Utica, N. Y., due to tuberculosis, for each of the twenty consecutive years beginning with 1887 were as follows: 13.7, 13.9, 15.1, 14.2, 16.7, 13.5, 13.2, 13.6, 14.8, 12.6, 12.9, 12.1, 11.5, 9.2, 11.9, 10.1, 9.5, 8.9, 9.7, 9.5. Draw a curve to show the change in death rate due to this disease.

1. The first part of the document is a list of names and addresses, which are arranged in a table-like format. The names are listed in the first column, and the addresses are listed in the second column. The names are: John Doe, Jane Smith, and Bob Johnson. The addresses are: 123 Main St, 456 Elm St, and 789 Oak St.

2. The second part of the document is a list of names and addresses, which are arranged in a table-like format. The names are listed in the first column, and the addresses are listed in the second column. The names are: John Doe, Jane Smith, and Bob Johnson. The addresses are: 123 Main St, 456 Elm St, and 789 Oak St.

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STATION.	6	7	8	9	10	11
Bismarck, N. D.	4	12	8	4	12	4
Boston, Mass.	14	6	18	32	4	12
Chicago, Ill.	12	22	20	8	18	22
Denver, Col.	4	4	4	4	4	4
Galveston, Tex.	4	16	10	14	14	4
Helena, Mont.	6	6	18	12	6	6
New Orleans, La.	6	4	12	6	6	4
New York, N. Y.	11	7	18	36	6	9
San Francisco, Cal.	4	10	6	4	4	4
Winnipeg, Manitoba. . . .	14	0	18	12	32	4

16. The rainfall, in inches, at Buffalo, N. Y., on the six consecutive days beginning April 6, 1908, was .46, .02, .20, .14, 0, and .14. Represent this graphically.

17. The accompanying table gives the percentages of all the deaths in Utica, N. Y., in 1906, that were due to consumption, classified according to age. Represent the relation between the age and death rate graphically.

Age	Percentage
1 to 14	1.9
15 to 44	20.3
45 to 64	7.7
65 to over	1.2

18. The number of persons on the pension roll of the United States in various years has been as follows: in 1863, 14,791; in 1864, 51,135; in 1865, 85,986; in 1866, 126,722; in 1870, 198,696; in 1875, 234,821; in 1880, 250,802; in 1885, 345,125; in 1890, 537,944; in 1895, 970,524; in 1900, 993,529; in 1905, 998,441; in 1907, 967,371. Show, by a curve, the growth of the pension system of the United States.

19. The following figures give the temperature at intervals of three hours of a typhoid fever patient in a New York hospital, beginning at 8 P. M., and running for 11 days:

102.8, 102.2, 102, 101.5, 101.4, 99.6, 101.5, 101.8, 102, 102, 102.4, 101.8, 101, 99.6, 100.6, 102.7, 102.3, 102.5, 101.5, 101, 100.7, 99.8, 100.5, 102.4, 102.2, 102.2, 101, 101, 100.4, 99.3, 100, 101.5, 102.5, 102.2, 101.3, 100.7,

100.4, 100, 100, 101.5, 102, 101.8, 101, 100.5, 100, 99.5, 100, 101.8, 102, 102, 101.8, 100.5, 99.5, 100, 101, 101.5, 101.2, 100.3, 99.3, 99.6, 98.7, 98.4, 100.5, 101.8, 101.8, 100, 99.3, 99.3, 99, 99.5, 100.8, 101.8, 100.5, 99.4, 99, 99, 98, 98, 99.8, 100, 99.5, 99, 98, 98, 98, 98.3, 99, 99.2.

Trace the rise and fall of temperature by means of a curve.

20. The following is the temperature record of a patient who recovered from scarlet fever, the record being made every morning and every evening: 99, 103, 101.8, 101.5, 100.1, 101, 100, 100, 99.8, 100.3, 99, 100, 98.9, 99.1, 98.8, 98.9. Represent the record by a curve.

21. The following is the temperature record of a patient who died with scarlet fever, the record being made every morning and evening: 100.5, 103.9, 102.5, 104.6, 103, 105, 103.8, 105.2, 103, 105, 104.4, 104.8, 105.6, 103.5, 105.2, 103.8, 104.6, 106.8. Represent the record by a curve.

PART IV

SUPPLEMENTARY VOCATIONAL PROBLEMS

The following problems are types of the problems met in agriculture, the building trades, shop-work, and other vocations. They are given in a miscellaneous list, and from them those may be selected for supplementary work that should be of value to the students of particular communities.

1. A field of 30 acres yields 25 bu. of wheat per acre. The wheat is sold for 90 ct. per bushel. The cost of growing and marketing the crop is \$225. What is the net profit from the crop? What sum placed at 6% interest would yield the same profit?

2. A bushel of wheat contains 20 oz. nitrogen, 8 oz. phosphoric acid, 5 oz. potash; a bushel of potatoes contains 3 oz. nitrogen, 1 oz. phosphoric acid, 4 oz. potash. If a field yields 27 bu. of wheat per acre, or 120 bu. of potatoes per acre, how many pounds of each of the three fertilizers are consumed in producing an acre of wheat and in producing an acre of potatoes? Which crop is the harder on the soil?

3. Loam soil contains 7 lb. nitrogen, 3 lb. phosphoric acid, and 8 lb. potash per ton. If the soil is cultivated to a depth of 8 inches, how many pounds of each of these three fertilizers in an acre of cultivated soil, a cubic foot of soil weighing 75 lb.? How many crops of wheat, 25 bu. to the acre, would be required to exhaust the phosphoric acid from the cultivated soil?

4. The average yield of a certain field is 36 bu. of corn per acre. By putting in an average of 2 days to the acre more time

in cultivation of the crop, the farmer is able to increase the yield to 54 bu. per acre. If corn is worth 45 cents per bushel, how much per day does he get for his extra work?

5. A farmer is able to grow only 2 tons of marsh hay per acre on land before it is tilled, and raises 50 bu. of corn per acre on it after it is tilled. If the hay is worth \$4.50 per ton, and the corn 40 cents per bushel, what is the increase in the value of the crop due to drainage?

6. A field of corn is planted in rows 44 in. apart each way. How many hills are there to an acre? What should be the yield, if the average hill produces 3 ears of a size such that 120 make a bushel?

7. If a cow eats pasture worth \$6 per year, $3\frac{1}{2}$ tons of hay worth \$6.50 per ton, and 1200 lb. of chopped feed worth 75 cents per 100 lb., raises a calf worth \$6, gives 3 quarts of milk per day for 280 days in the year which is worth 20 cents a gallon, and produces manure annually worth \$12, what is her profit to the farm for a year?

8. A dairy cow requires 1 lb. of protein to 6 lb. carbohydrates in her food. Dry peas contain 10 lb. of protein and 32 lb. of carbohydrates per bu. (60 lb.). Hay contains 88 lb. protein and 880 lb. of carbohydrates per ton. What should be the proportion of the quantities of dry peas and hay fed to a dairy cow, if these are to constitute her ration?

9. A farmer has a herd of 12 dairy cows that average 22 lb. each of milk per day. The milk tests 3.8% butter-fat, and butter-fat is worth 28 cents per pound. What is the daily income from the herd?

10. A sugar factory contracts to pay \$4.25 per ton for beets which test 14% or less of sugar, and to pay 20 cents extra per ton for each additional per cent above 14%. A man has 12 acres of beets yielding 15 tons per acre that test 14%, and 10 acres

yielding $13\frac{1}{2}$ tons per acre that test 15% sugar. What is the value of his crop? If he spends \$400 for labor to produce the crop, what is his average net profit per acre?

11. Make out the following monthly account of a farmer in the proper form, and balance it. His receipts were: Sale of 400 bu. of corn at 35 ct., 1200 bu. of wheat at 94 ct., 1600 lb. milk at 80 ct. per 100 lb., 20 doz. eggs at 16 ct. per doz. His expenses were: Hired help \$324, fertilizer \$65, wire for fencing \$12.25, threshing \$85.65, set of harness \$32.40, and groceries \$31.75.

12. What is the cost of paving and curbing a street 2640 ft. long and 42 ft. wide, if the contract price is \$1.25 a square yard for paving and 45 cents a linear foot for curbing?

13. A bin is to be constructed to occupy a space 10 ft. long and 6 ft. wide, and is to hold 9 tons of coal (2,000 lb. to a ton). If there are 64 lb. in a cubic foot of coal, what must be the depth of the bin?

14. A painter wishes to raise a ladder to the roof of a building which is 72 ft. high. It is necessary to have the ladder project beyond the eaves 3 ft. and have the foot placed 12 ft. from the wall of the building. What is the required length of the ladder?

15. A basin 50 ft. in diameter is surrounded by a walk 10 ft. wide. How many cubic yards of gravel will it take to cover the walk to a depth of 4 inches?

16. In a rectangular piece of land 120 ft. long and 70 ft. wide a cellar is to be dug for a building. The excavation is to be 42 ft. long, 36 ft. wide, and 8 ft. deep. The earth excavated is to be placed on the surrounding ground. What will be the depth of the filling? If the level of the lawn is to be raised 3 ft. in all, how many cubic yards of earth must be bought and hauled from elsewhere?

17. The surface of a pond contains 3 acres of commercial ice. The water is frozen so that the ice is fifteen inches thick. If ice is 0.93 as heavy as an equal volume of water, find the weight of the crop of ice in tons.

18. A cylindrical water tank is built to hold exactly 220 gallons. The height of the tank is 6 feet. What is the diameter in inches? (1 gal. = 231 cu. in.)

19. A man wishes to put a water tank in his attic. The tank is 28 in. long, 24 in. wide, and allows 20 in. for the depth of the water. How many gallons will the tank hold?

20. The excavation for a building is to be 48 ft. by 84 ft. and 6 ft. deep. Allowing 6 in. on all four sides to work in, beyond the external surface of the brick-work; find the amount of the excavation.

21. A cellar 50 ft. long, 30 ft. wide, and 7 ft. deep was excavated by 5 men in 8 days. At a contract price of 22 cents a cubic yard, what did each man earn daily?

22. A cellar wall 16 in. thick, 50 ft. long, and 7 ft. high is made of concrete (1 part Portland cement to 8 parts gravel). Estimate the cost of material when cement is \$1.80 per barrel (400 lb.) and gravel 80 cents per load (1 cu. yd.). One cu. ft. of dry cement weighs 150 lb.

23. When cement costs \$1.80, sand 16 cents, and gravel 10 cents, per barrel, estimate the cost of material to cement the floor of a cellar 40 ft. by 60 ft. A base 3 in. thick, made of 1 part cement and 8 parts gravel, is first laid; then upon this is spread a wearing surface 1 in. thick, made of 1 part cement and 1 part sand. One barrel of cement used with 8 barrels of gravel will cover 300 sq. ft. one in. thick, when used with one barrel of sand it will cover 68 sq. ft. 1 inch thick.

24. An architectural drawing of a house is made to the scale of $\frac{3}{4}$ in. to the foot. Find the number of square inches occupied

on the drawing by a lot of land which is 120 ft. long and 80 ft. wide.

25. Flooring has a $\frac{3}{4}$ -inch "tongue and groove," and is made in 3 in., 4 in., 5 in., and 6 in. widths. How much should be added to the area to be covered to find the number of board feet required for each of these widths?

26. At \$62 per M, find the cost of 3-inch oak flooring for a floor space 30 ft. by 45 ft. Add $\frac{1}{4}$ for "tongue and groove."

27. Make estimates for the floors of 3 rooms, each 18 ft. by 22 ft., of the following material:

3-inch oak flooring at \$62 per M. Add $\frac{1}{4}$.

3-inch maple flooring at \$48 per M.

4-inch pine flooring at \$35 per M. Add $\frac{1}{4}$.

6-in pine flooring at \$30 per M. Add $\frac{1}{4}$.

28. Weather-boarding is laid with a lap of from $\frac{3}{4}$ to $1\frac{1}{4}$ inches. A contractor estimates the surface, deducts for openings, and adds $\frac{1}{4}$ for the lap. At \$26 per M for poplar siding, find the cost of the siding for a building 320 ft. around and with an average height of 32 ft. Allow for 3 doors $3\frac{1}{2}$ ft. by 7 ft. and 8 windows 3 ft. by 6 ft.

29. Four inches is the standard width of a shingle. They are laid usually $4\frac{1}{2}$ inches "to the weather." They are sold in bunches of 250 each. Find the cost of red cedar shingles for two sides of a roof, each 20 ft. by 48 ft., the lower tiers being laid two shingles deep, at \$4.50 per M.

30. If shingles of an average width of 4 inches are laid with 5 inches to the weather, compute the number per square (100 sq. ft.).

31. Estimate the cost of the material for lathing and plastering the walls and ceiling of a room 30 ft. by 40 ft. and 13 ft. high, if 1 ton of hard wall plaster, costing \$6.50, will cover 100

sq. yd., and 1 bundle of laths (50), costing \$6 per M, will cover 4 sq. yd. (No deductions are made by plasterers for openings.)

32. In estimating the cost of painting a building, the contractor estimates that 1 gal. of paint will cover 250 sq. ft. and that the labor will cost twice as much as the material. What shall a contractor ask for painting a house two coats, the perimeter being 180 ft. and the average height 20 ft., if paint costs him \$1.40 per gal. and he wishes to make 20 % profit on material and labor?

33. If $1\frac{1}{4}$ barrels of lime are required for plastering 50 sq. yd., how many barrels would be required for a job of plastering equivalent to wall surface 150 ft. by 12 ft.?

34. An architect, in the specifications for a building, requires that the flooring shall not contain more than 10% of moisture, *i. e.*, 10 % of its weight in water. From the flooring delivered at the building average pieces were selected and weighed, then dried in an oven for several hours where the temperature was greater than boiling, then weighed again. The weight before drying was 3 lb. 6 oz., and after drying, 3 lb. 2 oz. Were the specifications complied with?

35. Lumber used in a building was tested for moisture, as in Prob. 34. The weight before drying was 5 lb. 12 oz., and after drying, just 5 lb. What was the per cent. of moisture?

36. Soft pine shrinks approximately $\frac{3}{4}$ in width by drying in open air. How many board feet must be allowed for shrinking in buying soft pine flooring in a green state for a room 24 ft. wide and 36 ft. long?

37. How long must the carriage for a stair be cut, if the distance between the floors is 12 ft., and if the height of a riser is to be 8 in. and the depth of a tread 10 in.?

38. Oak, in seasoning, shrinks about 3.1% across grain, but practically none lengthwise. How much will 3250 feet of green oak make when seasoned?

39. Short cast-iron posts, with a length in inches not exceeding 8 times their least dimensions, may be safely loaded (will support a weight without bending or breaking) with 6 tons for each square inch of cross-section. Find the safe load of such a post that is solid and 6 in. in diameter; of one that is solid and 12 in. in diameter.

40. Find the safe load of a short hollow cylindrical cast-iron post 10 in. in outer diameter and 1 in. thick.

41. The safe load (total weight that can be supported) of a rectangular wooden beam, supported horizontally at both ends and uniformly loaded over the entire span (length) is $\frac{2 \times \text{breadth} \times \text{square of depth} \times K}{\text{span in feet}}$, the breadth and depth being taken in inches. For white oak K is 75; for white pine, 65; for spruce, 70. Compute the safe loads of beams as follows:

A white oak beam, 12 ft. long, breadth 2 in., depth 6 in.

A white pine beam, 16 ft. long, breadth 3 in., depth 8 in.

A spruce beam, 20 ft. long, breadth 4 in., depth 12 in.

42. The strength of a rectangular beam varies directly as its breadth multiplied by the square of its depth. Compare the strength of a beam 6 in. in breadth and 4 in. in depth with the strength of a beam 4 in. in breadth and 6 in. in depth, both being of the same material.

43. The Chicago building ordinance, 1893, gave 3500 lb. per sq. ft. of surface as the bearing power of pure clay soil 15 ft. deep, without admixture of any foreign substance except gravel. What may be the dimensions of the foundation of a pier that is to support 12 tons weight?

44. Pure dry sand will support a load of 4000 lb. per sq. ft. What safe weight will a foundation $3\frac{1}{2}$ ft. by $5\frac{1}{2}$ ft. support on pure dry sand?

45. The safe weight, or working load, that masonry of hard brick and lime mortar will support is about 6 tons per sq. ft. A brick pier 5 ft. long must be how wide to support a weight of 65 tons?

46. Plumbers, to find the pressure exerted at any point on the system of plumbing, multiply the vertical distance of the point below the highest point of water connected with the system by .434. This gives the pressure in pounds per square inch. Find the pressure at a point of the plumbing 50 ft. below the water level; 35 ft.; 5 ft.; 160 ft.; 240 ft.

47. Compute the amount of the following portion of a plumber's estimate:

20 ft. of 2-in. Pipe at \$0.35				
60 ft. of 4-in. " " 0.80				
8 2 in. Bends " 0.50				
2 4-in. Y's " 0.90				
Discount 25 and 5%				

48. In building construction the distance apart, in feet, at which white pine floor-beams are to be placed to support the weight in ordinary dwellings, is computed by the following rule: Multiply the cube of the depth of a beam by its breadth, both in inches, and divide the product by the cube of the length in feet multiplied by 0.65. A span of 17 ft. in a dwelling is to be covered by white pine beams 3×12 inches. At what distance apart should they be placed? What space if the beams are 2×12 inches?

49. A car 36 ft. long and 8 ft. 6 in. wide, loaded to a depth of 5 ft., contains how many bushels of wheat? (1 cu. ft. = .8 bu.) What is the weight of the load at 60 lb. per bushel?

50. How much paint will it take to cover the outside of a

railroad water tank, including top and bottom, if the tank is 26 ft. in diameter and 14 ft. high, allowing 2 qt. of paint per square (100 sq. ft.)?

51. A locomotive is extended 3 ft. in order to gain heating surface and thus increase the steaming capacity. If there are 875 2-inch tubes, by how many square feet is the heating surface of the tubes increased?

52. Find the pressure tending to blow the head out of an air drum 16 in. in inside diameter when the gauge shows a steam pressure of 90 lb. per square inch.

53. What is the propelling force exerted against the piston in a locomotive when the steam pressure is 100 lb. per square inch, the inside diameter of the cylinder being 36 inches?

54. A drum head 26 in. in diameter is cut from a sheet of $\frac{1}{2}$ -in. steel plate 27 in. square. Find the weight of the portion cut off. ($\frac{1}{2}$ -in. plate weighs 20.4 lb. per sq. ft.)

55. A car 36 ft. long and 8 ft. 6 in. wide contains 1000 bu. of wheat. If the wheat is leveled off, how high does it reach in the car? (1 cu. ft. = .8 bu.)

56. A box car with 80,000 lb. capacity is 36 ft. long, 8 ft. 6 in. wide, and 8 ft. high. What weight of oats will it carry if it is filled? (Oats weigh 32 lb. per bu.)

57. To what depth may the car described in Problem 56 be filled with wheat not to exceed its capacity of 80,000 lb.? (1 bu. weighs 60 lb.)

58. May the same car be filled with Syracuse salt weighing 56 lb. per bushel?

59. What is the greatest weight of grass seed that the same car will carry? (1 bu. weighs 14 lb.)

60. A track tank 1450 ft. long and 19 in. wide, filled to an average depth of $4\frac{3}{4}$ in., contains how many gallons of water?

61. How many pieces 30 in. by 70 in. can be cut from a piece of sheet-iron 100 in. by 210 in., with no allowance for waste? Make a sketch showing the lines along which the sheet should be cut.

62. A grindstone when new is 5 ft. in diameter and has a 12-inch face. What has it lost in weight when worn down 3 inches? One cubic foot of sandstone weighs 150 lb.

63. A locomotive has a netting in the smoke box 24 in. by 24 in., made of wire .125 in. in diameter. If the netting is $2\frac{1}{2}$ in. by $2\frac{1}{2}$ in. mesh, what part of the opening is taken up by the wire? (By $2\frac{1}{2}$ in. mesh is meant $2\frac{1}{2}$ wires to the inch.)

64. The locomotive described in Problem 63 has 400 tubes of $1\frac{1}{2}$ in. inside diameter. What is the ratio of the opening through the tubes to the free opening (area of smoke box less the space taken by the wires) of the smoke box?

65. Find the capacity in gallons of a tank 10 ft. in diameter and 14 ft. deep.

66. How deep shall one make a tank 12 ft. in diameter so that it may hold 4000 gallons?

67. Find the weight of an iron pipe 14 ft. long, 15 in. external diameter, and 14 in. internal diameter. (1 cu. in. of iron weighs 0.27 lb.)

68. A railway embankment is 15 ft. high, the top is 28 ft. wide, and the sides have a slope of 1 in 2 to the horizontal. How many cubic yards of earth are needed for 150 feet of the embankment?

69. The pressure on the end of a rectangular tank that is full of water equals one-half of the weight of a rectangular volume of water whose base is the area of the end and whose height is the depth of the tank. Find the pressure on the end of a tank 6 ft. wide and 5 ft. high.

70. A vertical masonry dam in the form of a rectangle 180

ft. long and 40 ft. high must withstand how much pressure?
(See Prob. 69.)

71. When the wind strikes a flat surface at right angles the pressure upon the surface exposed, in pounds per square foot, is found by multiplying the square of the velocity in miles per hour by .0054. Find the total pressure to which a house is exposed if it is 48 ft. long and 22 ft. high, if it is struck by the wind at right angles with a velocity of 30 mi. per hour.

72. What is the pressure of the wind upon a sail which exposes 90 sq. ft. to the wind, when wind blowing at the rate of 18 mi. per hour strikes it at right angles?

73. How many revolutions per minute ought a piece of cast-iron 4 inches in diameter be allowed to make in a lathe, if the cutting speed is usually 40 feet per minute?

74. How long should the wooden pattern be made for a brass casting which is 3 ft. 4 in. long, if the shrinkage allowance of brass when it cools is $\frac{3}{16}$ inch per foot?

75. If a horizontal boiler has 96 2-inch tubes, what should be the diameter of the chimney to properly carry off the smoke and gas, if the rule calls for a chimney equal in sectional area to the combined sectional area of the tubes?

76. A receipt for a blue print solution is as follows: 2 oz. citrate of iron and ammonia, 1.33 oz. red prussiate of potash, 16.5 oz. water. Give proportions if 3 oz. of citrate of iron and ammonia are used.

77. A fly wheel of an engine is 10 ft. in diameter and makes 200 revolutions per minute. It is directly connected by a belt to a wheel 3 ft. 4 in. in diameter. What is the speed of the latter wheel?

78. What must be the diameter of the driven pulley in order that it may make 375 revolutions when belted to a 30-inch driver making 225 revolutions?

79. The composition of one kind of Babbitt's metal is 4 parts copper, 8 parts antimony, and 88 parts tin. Find the amount of each material necessary to produce 175 pounds of the metal.

80. Give the relation between the carrying capacities of one 8-inch pipe and two 4-inch pipes when each of the pipes discharges water with a velocity of 75 ft. per minute.

81. Square pieces of tin, 8 in. on a side, are fed to a stamping machine which has 4 dies cutting out four of the largest possible circular pieces of tin. What is the area of one of these discs?

82. If iron is 7.8 times as heavy as water, find the weight of a block of iron 15 in. long whose cross section is in the form of a right triangle, the hypotenuse of which is 5 in., and the base 3 in. (1 cu. ft. of water weighs 62.5 lb.)

83. A bar of zinc 28 in. long, $2\frac{1}{4}$ in. wide, and $1\frac{1}{2}$ in. thick, is melted into the form of a cube. What will be its new dimensions?

84. Find the heating surface of the 291 tubes of the Pacific type of locomotive when each tube is 19 ft. 6 in. long and $2\frac{1}{2}$ in. in diameter.

NOTE.—The following problems represent a type frequently met in shop work, the building trades, etc., where the values of quantities are computed by means of *formulas*. A formula expresses a rule in symbols by means of letters.

85. The indicated horse-power of an engine is represented by the formula $H. P. = \frac{p \times l \times a \times n}{33,000}$, where p = the mean effective pressure in pounds per square inch, l = the length of stroke in feet, a = area of piston in square inches, n = twice the number of revolutions. Compute the H. P. of an engine in which a test showed p to be 35 lb. per sq. in., the number of revolutions 60, while the area of the piston was 706.8 sq. in., and the length of stroke 48 in.

SOLUTION. Using the values of p , l , a , and n , for this engine in the formula, we have, indicated, $H.P. = \frac{35 \times 4 \times 706.8 \times 120}{83,000}$. Performing the multiplications and division indicated, (using cancellation), or computing by logarithms, we get $H.P. = 359.8$.

86. The horse-power that can be transmitted by a shafting with a working stress of 5,000 lb. per sq. in. is $H.P. = \frac{n \times d^3}{64}$,

where n = number of revolutions per min., d = diameter of shafting in inches. How many horse-power can be transmitted by such a shafting 4 in. in diameter, making 76 revolutions per minute? One 5 in. in diameter, making 100 revolutions per minute?

87. The stress P , in pounds per sq. in., due to centrifugal force in the rim of a fly-wheel is found by the formula $P = \frac{W \times V^2}{32.2 R}$, where W = weight of 1 ft. length of rim 1 sq. in.

in cross section, V = velocity of rim in feet per sec., R = radius of rim in feet. In a cast-iron wheel, $W = 3.1$ lb. Find the stress in the rim of a cast-iron fly-wheel 8 ft. in diameter, running at 160 revolutions per minute.

88. The formula for tie-rods for beams supporting brick arches is: Horizontal thrust in pounds per linear foot of arch = $\frac{1.5 \times W \times S^2}{R}$, where W = weight on arch in pounds per

sq. ft., S = span of arch in feet, R = rise of arch in inches. Find the strain on the tie-rod in an arch on which the weight is 360 lb. per sq. ft., the span 4 ft., and the rise of the arch 18 in.

89. If D inches is the deflection in the middle of a beam supported at both ends and loaded in the middle, then $D = \frac{W \times L^3}{48 \times E \times I}$, where W = weight of load in pounds, L = length of beam in inches, and E and I are numbers depending upon

the form and material of the beam. In a wrought-iron bar 2 in. deep and 1 in. wide $I = \frac{2}{3}$ and $E = 29,000,000$. If the bar is 6 ft. long, find the deflection due to a load of 3200 lb.

90. If a beam L ft. in length is supported at both ends and loaded uniformly throughout its length with W lb. per foot, the maximum deflection D , in inches, is obtained by the formula $D = \frac{5 \times W \times L^4}{384 \times E \times I}$. If a beam in which $E = 30,000,000$ and $I = \frac{5}{8}$ is 12 ft. long, and the load 200 lb. per ft., find the maximum deflection.

91. In an electric current, R , the combined resistance of the resistances r_1, r_2, r_3 in parallel, is found by the formula $R = \frac{r_1 \times r_2 \times r_3}{r_1 \times r_2 + r_1 \times r_3 + r_2 \times r_3}$. What is the combined resistance of resistances of 5.2 ohms, 6.4 ohms, and 7.6 ohms, put in parallel?

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A BOOK OF ANSWERS
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COMMERCIAL AND INDUSTRIAL
FOR
HIGH SCHOOLS, INDUSTRIAL SCHOOLS, COMMERCIAL
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BY
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MICHIGAN STATE NORMAL COLLEGE, YPSILANTI, CO-AUTHOR OF THE SOUTHWORTH-
STONE ARITHMETICS, THE STONE-MILLIS ALGEBRAS, ETC.

AND
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Page 10. — 1. 57,806. 2. 61,058. 3. 60,349. 4. 65,839. 5. 61,059. 6. 64,702. 7. 76,092.

Page 11. — 1. 237. 2. 1155. 3. 2139. 4. 18,131. 5. 170,927. 6. 1,609,125. 7. 16,461,548. 8-33. (a) 153,264. (b) 358,783. (c) 979,737. (d) 344,518. (e) 797,892. (f) 713,236. (g) 751,820. (h) 606,345. (i) 923,814. (j) 548,479. (k) 473,235. (l) 832,748. (m) 726,236. (n) 724,930. (o) 1,094,675. (p) 745,317. (q) 1,058,727. (r) 951,103. (s) 676,213. (t) 484,432. (u) 650,523. (v) 424,606. (w) 1,039,644. (x) 406,887. (y) 1,007,684. (z) 788,414.

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Pages 20, 21. — 1. 134 484,741,673. 2. 31,689,423,737. 3. 34,318,075,319,517. 4. 11,624,276,694,010. 5. $121\frac{1}{2}$. 6. $145\frac{1}{2}$. 7. $172\frac{1}{2}$. 8. $243\frac{1}{2}$. 9. $157\frac{1}{2}$. 10. $224\frac{1}{2}$. 11. $206\frac{1}{2}$. 12. $842\frac{1}{2}$.

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- Page 35. — 4. .4198+. 5. 20.8863+. 6. 1.807. 7. .0505+.
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42. $\frac{5}{8}$. 43. $1\frac{1}{2}$. 44. $\frac{3}{10}$.

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14. \$1.66; \$4.16.

Pages 56-60. — 1. \$46.51. 2. Yes; \$8.80. 3. \$10.63. 4. \$31.85; \$32.50. 5. \$73.75; \$75.26. 6. \$61.02. 7. Incorrect. 8. \$188.70;

Pages 60-62. — (*Oral.*) 1. 12½ ct. 2. 31½ ct. 3. 25 ct. gained on each. 4. 28 ct. 5. 26%. 6. 8%. 7. 10%. 8. 12%. 9. 60 ct. 10. 56½%. 11. 33½%. 12. 66½%. 13. \$700; 20%. (*Drill exercises.*) 1. \$.70; \$4.20; \$3.78; \$.28 gain. 2. \$1.90; \$9.50; \$8.32; \$.72 gain. 3. \$3.50; \$14.00; \$13.65; \$3.15 gain. 4. \$1.20; \$4.80; \$4.32; \$.72 gain. 5. \$1.60; \$11.20; \$10.64; \$1.04 gain. 6. \$3.28; \$19.68; \$18.11; \$1.71 gain. 7. \$9.92; \$34.72; \$26.04; \$1.24 gain. 8. \$7.43; \$23.93; \$19.94; \$3.44 gain. 9. \$8.60; \$25.80; \$22.57; \$5.37 gain. 10. 20% gain. 11. 26% gain. 12. 16½% gain. 13. 25% gain. 14. 20% gain. 15. 5% gain. 16. 22½% gain. 17. 38% gain. 18. 4% gain. 19. 4% gain. 20. No loss or gain. 21. 5% gain. 22. 2½% loss. 23. 10½% gain. 24. 4½% gain. 25. 6½% gain. 26. 4% loss. 27. 20% gain.

Pages 62, 63. — 1. \$4500; \$2700. 2. \$60,000. 3. 10%. 4. \$2.10; \$105.

Page 63. — 1. \$18; 3½ ct. 2. \$7.20; 1 ct.; 65 ct. 3. \$250.

Pages 64, 65. — 1. \$17.50; 3½ ct. 2. \$246.17. 3. \$34.80. 4. \$561.15. 5. \$40.55; 8.55%. 6. \$327.20. 7. \$123.40. 8. \$415.88. 9. \$2403.97; \$379.97.

Page 66. — 1. \$3437.50; \$125. 2. \$1837.50; \$75. 3. \$168,175; \$175. 4. \$5880; \$280.

Page 70. — 2. \$312.50; \$5312.50. 3. \$402.67; \$6442.67. 4. \$731.25; \$8231.25. 5. \$302.72; \$9762.72. 6. \$48.75; \$1398.75. 7. \$121.10; \$8581.10. 8. \$48.60; \$4811.40. 9. \$63.19; \$6436.81. 10. \$19.25; \$1880.75. 11. \$43.72; \$3006.28. 12. \$46.04; \$5053.96. 13. \$15.60; \$1934.40.

Pages 71, 72. — 20. \$18.82. 21. \$16.80. 22. \$7.48. 23. \$13.86. 24. \$26.04. 25. \$28.88. 26. \$8.91. 27. \$13.44. 28. \$8.59. 29. \$27.36. 30. \$8.75. 31. \$7.97. 32. \$8.31. 33. \$9.30. 34. \$8.75. 35. \$11.45. 36. \$451.50. 37. \$121.00. 38. \$836.23. 39. \$654.88. 40. \$820.13. 41. \$971.20. 42. \$993.07. 43. \$744.31. 44. \$976.80.

Pages 73, 74. — 2. \$4.35; \$562.65. 3. \$6462.61. 4. \$808.88. 5. 2nd, \$7.50; 2nd, \$3.75. 6. \$533.07; \$534.60; \$537.39. 7. \$6.97; \$766.83.

Pages 76, 77. — 1. \$2.55. 2. \$5.34. 3. \$4.76. 4. \$7.13.
 5. \$8.84. 6. \$40.23. 7. \$77.47. 8. \$109.20. 9. \$161.97.
 10. \$90.16. 11. \$2.90. 12. \$5.44. 13. \$6.64. 14. \$16.56.
 15. \$17.71. 16. \$19.69. 17. \$19.23. 18. \$6.53. 19. \$7.58; \$442.42.
 20. \$26.56; \$933.44. 21. \$55.18; \$4244.82. 22. \$33.92; \$1816.08.
 23. \$41.17; \$3758.83. 24. \$52.42; \$4197.58.

Pages 77, 78. — 1. \$864.27. 2. \$1552.38. 3. \$1516.61.
 4. \$1538.27.

Page 80. — 2. \$9989.08. 3. \$6848.09. 4. \$9913.33. 5. \$23,395.64.
 6. \$2182.45.

Pages 83-87. — (*Oral.*) 1. \$135.25; \$1352.50. 2. \$86.50; \$43.25;
 \$432.50. 3. \$163.50; \$1635; \$1.25 for 10. 4. \$16; \$160. 5. 6%; \$60.
 6. 8%; \$200. 7. \$200. 8. \$500; \$2000. 9. 50; \$6000.
 10. 20; \$1840. 11. 10%; 4%. 12. 8%. 13. \$200; 199 $\frac{1}{2}$. 14. 50.
 15. \$120; \$20; 16 $\frac{1}{2}$ %. 16. \$10 per share; \$47.50. 17. 74 $\frac{1}{2}$.
 18. 81 $\frac{1}{2}$. (*Written.*) 1. \$4425; \$2815; \$1370. 2. 3.5% approx.; 6.2%.
 4. \$1037.50. 5. \$11,312.50. 18. \$6265; \$400; 6.38%. 19. \$2664.
 20. \$811.46; \$2743.10. 21. 3.64%. 22. \$1415.50.

Pages 89-91. — (*Oral.*) 1. \$975; \$1000. 2. \$9000. 3. \$150. 4. \$500.
 5. 30. 6. \$3 per \$100 bond. 8. \$6; 7 $\frac{1}{2}$ %. 9. The first. (*Written.*)
 1. 9.38%. 2. 7.22%. 3. 5.16%. 4. The first by 4.97% — 4.17%.
 5. 4.95%; 6.42%. 6. \$2437.50; \$100; 4.05%; 4.39%; 4.51%.
 7. \$45; \$382.41 including brokerage of $\frac{1}{2}$ % each.

Pages 92, 93. — 1. Yes. 2. Yes. 3. \$302.87. 4. \$202.67.
 5. \$36.13. 6. Nearly \$1560. 7. \$59,060.45; \$2400.
 (He could buy 60 \$1000 bonds and have \$260.45 left, not counting
 brokerage.) 8. \$32,649.63; about \$1600.

Pages 94, 95. — 1. 11 yr., 3 mo. 2. \$66.87. 3. 10 yr., 11 mo.
 4. 11 yr., 1 mo.

Pages 98-101. — 1. \$8.60. 2. \$18.01; \$135.08. 3. 17.5%; \$263.40.
 4. \$94.40. 5. 25%; 17.2%. 6. \$7.76. 7. 1st, \$798.96; 1st, \$470.63;
 1st, \$29.40; 2nd, \$563.59. 8. \$3391.54 more in bank. 9. \$10,727.86;
 \$727.86. 10. \$5147.95 more in insurance. 11. \$25.17.
 12. \$332.76. 13. \$11,461.07. 14. \$1192.03; \$192.03; \$9.60.
 15. \$16.38; \$248.60; \$175.35. 16. \$8957.12. 17. \$6372.41 more in
 insurance; \$2944.04 more in bank; \$16,734.68 more in bank.
 18. About \$92.70.

Pages 108-110.—9. July 5; 56 da. 10. \$344.75. 11. \$343.88.
 14. \$493.75. 15. \$490. 16. \$10.25; first. 18. \$151.14.
 19. \$147.50. 20. \$114.17; \$114.57. 21. \$197.45. 22. \$100.31
 23. \$80.97; \$.96 more.

Page 113.—1. 1329.3. 2. 791.3. 3. 197.8. 4. 59.5. 5. 1826.5.
 6. 84.3. 7. 1523.5. 8. 125.9. 9. 137.2. 10. 6672.0. 11. 2.2.
 12. 126.1. 13. 8.8. 14. 22.5. 15. 241.2. 16. 53.4. 17. 7.5.
 18. 4.3. 19. 81.5. 20. 49.1.

Page 115.—1. 3. 2. 5. 3. 9. 4. 3. 5. 4. 6. 6. 7. 7.
 8. 8. 9. 10. 10. 5. 11. 6. 12. 3. 13. 2. 14. 7. 15. 2.
 16. 10. 17. 15. 18. 8. 19. 35. 20. 6. 21. 4. 22. 30.
 23. 20. 24. 45. 25. 3. 26. 2. 27. .1. 28. 1.2. 29. .8.
 30. 2.5. 31. .2. 32. 1.2. 33. .9. 34. .1. 35. 2.

Page 116.—1. $\frac{2}{3}$. 2. $\frac{1}{4}$. 3. $\frac{3}{4}$. 4. $\frac{5}{7}$. 5. $\frac{8}{10}$. 6. $\frac{3}{4}$. 7. $\frac{4}{15}$.
 8. $\frac{1}{3}$. 9. $\frac{2}{3}$. 10. $\frac{1}{4}$. 11. $\frac{5}{8}$. 12. $\frac{1}{7}$. 13. $\frac{2}{3}$. 14. $\frac{1}{5}$. 15. $\frac{3}{10}$.
 16. $\frac{1}{2}$. 17. $\frac{2}{3}$. 18. $\frac{1}{2}$. 19. $1\frac{1}{2}$. 20. $1\frac{1}{2}$.

Page 120.—1. 28. 2. 43. 3. 63. 4. 39. 5. 73. 6. 82.
 7. 234. 8. 203. 9. 416. 10. 702. 11. 185. 12. 3.5. 13. 12.1.
 14. 3.59. 15. 11.2. 16. 2.15. 17. 51.1. 18. .162. 19. .236.
 20. .0418. 21. 1.732. 22. 2.236. 23. 3.464. 24. 2.645. 25. 1.414.
 26. .556. 27. 2.489. 28. .244. 29. 5.916. 30. 1.019. 31. .816.
 32. .935. 33. .408. 34. .845. 35. .522. 36. 866. 37. .559.
 38. .349. 39. .663. 40. .727. 41. .089. 42. .834. 43. .384.
 44. .554.

Page 122.—1. 47. 2. 14. 3. 25. 4. 63. 5. 83. 6. 698.
 7. 806. 8. 3.9. 9. 6.9. 10. 3.45. 11. 1.25. 12. 2.15. 13. 4.73.
 14. 1.54. 15. .53.

Pages 123, 124.—1. $\log_2 16=4$. 2. $\log_2 128=7$. 3. $\log_3 9=2$.
 4. $\log_5 125=3$. 5. $\log_4 64=3$. 6. $\log_7 2401=4$. 7. $\log_{10} 10000=5$.
 8. $\log_{10} 10=1$. 9. $\log_5 15625=6$. 10. $\log_6 216=3$. 11. $\log_{20} 400=2$.
 12. $\log_{12} 1728=3$. 13. $2^5=32$. 14. $3^2=9$. 15. $2^4=16$. 16. $4^2=16$.
 17. $3^3=27$. 18. $4^3=64$. 19. $5^2=25$. 20. $10^3=1000$. 21. $10^4=10$.
 22. $6^3=216$. 23. $12^2=144$. 24. $7^3=343$. 25. 4. 26. 9. 27. 5.
 28. 4. 29. 3. 30. 3. 31. 2. 32. 3. 33. 4. 34. 2. 35. 3. 36. 3.
 37. 4. 38. 6. 39. 12. 40. 3. 41. 2. 42. 3.

Page 125.—1. 2^{15} . 2. 4^{19} . 3. 6^{10} . 4. 2^{12} . 5. 5^9 . 6. 8^{15} . 7. a^{12} .
 8. a^{14} . 9. a^{13} . 10. a^{35} . 11. 4^4 . 12. 3^5 . 13. 5^3 . 14. 2^3 . 15. 7^3 .

16. 10^3 . 17. 6^5 . 18. a^5 . 19. a^4 . 20. a^1 . 21. a^3 . 22. 6^{13} .
 23. 2^{10} . 24. 5^3 . 25. 7^{24} . 26. 4^{120} . 27. a^{27} . 28. a^{35} . 29. a^{20} . 30. a^{84} .
 31. a^{14} . 32. a^{12} . 33. 2^4 . 34. 5^2 . 35. 3^3 . 36. 4^3 . 37. 9^2 . 38. 2^5 .
 39. 3^2 . 40. 4^3 . 41. 8^5 . 42. 10^5 . 43. 2^8 . 44. a^2 . 45. a^6 . 46. a^4 .
 47. a^7 . 48. a^8 .

Page 127. — 4. 1.2552. 5. 1.1761. 6. 1.3010. 7. 1.6990.
 8. 1.3801. 9. .3680. 10. .2219. 11. 1.8451. 12. 2.3222. 13. 3.
 14. .8239. 15. .6276. 16. .7385. 17. .7517. 18. .6078. 19. .2762.
 20. .3095 — .6555. 21. 2.2443 — 2.2980.

Page 131. — 1. 2.5145. 2. 2.8831. 3. 2.9661. 4. 2.2625. 5. 1.6304.
 6. .5955. 7. 9.4564 — 10. 8. .9675. 9. 1.7959. 10. 9.9253 — 10.
 11. 1.2989. 12. .9325. 13. 9.7875 — 10. 14. 8.9390 — 10. 15. 7.5065 —
 10. 16. 6.9609 — 10. 17. 2.9542. 18. 2.4150. 19. 2.6721. 20. 2.7782.
 21. .3010. 22. 1.5682. 23. 1.6902. 24. .8451. 25. 0. 26. 1.8325.
 27. 3.6532. 28. 4.5132. 29. 3.6321. 30. 3.9607. 31. 1.7983.
 32. 2.7216. 33. .4971. 34. 4.8150. 35. 1.4369. 36. .7261.
 37. 9.7984 — 10. 38. 7.8994 — 10. 39. 8.8048 — 10. 40. 3.6302.

Page 132. — 1. 21.1. 2. 3480. 3. 83700. 4. .587. 5. .00652.
 6. 354. 7. 4780000. 8. 2760. 9. 4.87. 10. 726. 11. .555.
 12. .0102. 13. 94.9. 14. .00686. 15. .0000551. 16. 4.06.
 17. 314.43. 18. 15.317. 19. 5483.7. 20. .5734. 21. 84.34.
 22. .02954. 23. 10352. 24. 2.975. 25. 44.67. 26. .06587.
 27. 997.5. 28. .2848.

Pages 133, 134. — 1. 15.3. 2. 649.85. 3. 10.231. 4. 5427.5.
 5. .05438. 6. 228.9. 7. .06248. 8. 392636000. 9. 3.27.
 10. 1.443. 11. .23927. 12. 3.385. 13. .09638. 14. 13.02.
 15. 79.63. 16. 31.835. 17. 445.3. 18. 7.88. 19. 12.217.
 20. .52975. 21. 7.013. 22. 9.024. 23. .0216. 24. 10.84.
 25. \$1128.20. 26. \$5012.22. 27. .987 in. 28. $D = 4.37$ in.

Pages 139, 140. — 1. 1,000 mm.; 1,000,000 mm. 2. 100 cm.;
 100,000 cm. 3. 25,000 m. 4. .00003 mm. 5. 370 m.
 6. 12.2 m.; 200 m.; 4 m. 7. .245 cm.; 300 cm.
 8. 4.2625 Hm.; 42625 cm.; .42625 Km. 9. .000425 sq. m.; 425 sq. mm.
 10. 250000 sq. cm.; 42.86 sq. cm. 11. .036284 sq. m.; .001234 sq. m.
 12. 286 sq. Dm.; 28600 sq. m.; 28600000 sq. dm.; 286000000 sq. cm.;
 28600000000 sq. mm. 13. .000001 cu. m.; .000000001 cu. m.
 14. 1000 cu. m.; 1,000,000 cu. m. 15. 26,814 cu. dm.; 26,814,000 cu. cm.;

26,814,000,000 cu. mm. 16. .005123 cu. Dm.; 5,123,000 cu. cm.
 17. 526.8 a; 52680 ca; 52680 sq. m. 18. 12000 cu. dm.; .012 cu. Dm.
 19. 8.2022; 10.7283. 20. 15.5; .506. 21. 0.9842; 19.685.
 22. 9.321. 23. 11.811; 7.874; 3.937.

Pages 143-147. — 1. $18\frac{1}{2}$. 2. 7 strips; $6\frac{1}{2}$ yd.; $46\frac{1}{2}$ yd.; \$58.33.
 3. 2200 sq. rd.; $13\frac{1}{2}$ A. 4. 5.51. 5. \$113.89. 6. \$3024. 7. 1920.
 8. \$30.21. 9. 1250. 10. 8800 yd. 11. \$1.08. 12. 7.88 bbl., approx.
 13. 107875 sq. yd. 14. 229375 sq. ft. 16. \$144.63. 17. \$44.55.
 18. 15833. 19. \$83.20. 20. 1002. 21. 4 hr., 12 min. 22. 3808 sq. in.
 23. \$19.20. 24. $7\frac{1}{2}$. 25. \$0.1495 per sq. ft. 26. (1) 30 bunches, or
 1500 laths; (2) \$9.00; (3) 24 bags wall plaster; (4) \$7.80; (5) \$22.58.

Pages 149-151. — 1. 2.54 ft., approx. 2. 720.3. 3. 5.4978 sq. in.
 4. \$61.09. 5. 2120.58 sq. in. 6. 57.55 ft. 7. 25133 mi.; 1047 mi.
 per hr. 8. $4\frac{1}{2}$ in. by $12\frac{1}{2}$ in. 9. 43.854 ft. 10. 1 ft. 11. 785.4.
 12. (1) 199; (2) 265; (3) 145. 13. (1) 327; (2) 191. 14. .97 ft.
 15. $16\frac{1}{2}$ in. 16. 300 revolutions per min. 17. (1) 10179 lb.; (2) 2545 lb.
 18. 47.124 sq. in. 19. 31.416 ft. per mi. 20. 75.4 ft. per sec.
 21. 20.3. 22. 9 in. 23. Capacity of 6-in. pipe twice that of other two.
 24. 121. 25. 15.49 in. 26. 51, approx.

Pages 152-155. — 1. 13 m. 2. 76.8 ft., approx. 3. 127.2 ft., approx.
 4. 63.7 ft., approx. 5. No. 6. 25.9 cm., approx.; 388.5 sq. cm.
 7. 207.4 ft. 8. 17.9 ft. 9. 207.6 yd. 10. 823.6 yd.
 11. 23498 sq. ft. 12. 125 lb. 13. 9 lb. 14. 5.83 ft. per sec.;
 120 yd. down stream. 15. 49.3 ft., approx. 16. 10.6 cm., approx.;
 56.25 sq. cm. 17. 740.3 mi. per hr. 18. 1.172 ft. 19. .62 in.
 20. 2.828 in. square.

Pages 155, 156. — (Oral.) 1. 8. 2. 8. 3. $4\frac{1}{2}$. 4. 3. 5. 9.
 6. 20. 7. 16. 8. 16. 9. $37\frac{1}{2}$. 10. 15. 11. 120. 12. 256.
 13. 216.

Pages 156, 157, Problems. — 1. 310 ft.; \$5.58. 2. \$11.57.
 3. 1920 ft. 4. 664 ft.; \$16.60. 5. \$370.50. 6. $2\frac{1}{2}$ in.; $\frac{1}{2}$.
 7. \$61.44. 8. \$122.03. 9. $\frac{1}{2}$. 10. \$24.62.

Page 158.

| | | | | | | | | | | |
|----------------------|----|----|----|-----|-----|-----|-----|-----|-----|-----|
| Diameter in inches | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 24 |
| Length in feet | 10 | 12 | 14 | 16 | 18 | 20 | 16 | 18 | 24 | 24 |
| Number of board feet | 40 | 61 | 88 | 121 | 162 | 211 | 196 | 263 | 384 | 600 |

Pages 160-163. — 1. 16 cm. 2. 36 cu. ft. 3. 5 m. 4. 1224 bu.;
 78440 lb. 5. 94 cm. 6. 249 cu. yd.; \$87.15. 7. \$95.41.
 8. \$84.38. 9. 3. 10. $33\frac{1}{2}$. 11. 514 $\frac{1}{2}$. 12. 248. 13. 1445 sq. ft.;
 94828 $\frac{1}{2}$ lb. 14. 350 lb. 15. 41.58 cu. in. 16. 4897.6 gal.
 17. 3456 cu. m. 18. 5802 cu. yd. 19. 32. 20. 1944.
 21. 81500 cu. ft. 22. 238.9 tons. 23. 496. 24. $5\frac{1}{2}$ in.
 25. 4.72 in., approx. 26. 1.75 ft., approx. 27. 1629 $\frac{1}{2}$ cu. yd.
 28. 452 $\frac{1}{2}$ lb.

Pages 165-168. — 1. 75.3984 sq. in.; 100.5312 sq. in.; 75.3984 cu. in.
 2. 646.1 sq. cm. 3. 4.59 in., approx. 4. 201.062 cu. in. 5. 23.56 cu. yd.
 6. 5229 cu. yd., approx. 7. 3225.26 gal., approx. 8. \$205.62.
 9. 1570.8 sq. ft. 10. .038 in. 11. 34.46 min. 12. .214 of it.
 13. .214 of it. 14. 1.29 in. 15. 42.02 in. 16. 7883.6 gal.
 17. 56.5488 sq. ft. 18. 219.912 sq. in. 19. 65.563 cu. ft.
 20. 3.45 gal. 21. 4778.5 gal. 22. 5618.228 gal.; 299.73 cu. ft.
 23. .276 lb. 24. 72.08 lb. 25. 52.6 lb. 27. 54287 gal.
 28. 8.8 in.; .6 in. 29. Large can holds $1\frac{1}{2}$ times as much as the other
 two. 30. 14.6 lb. 31. 11998 gal. 32. 104.78 lb.

Pages 169, 170. — 1. 4. 2. $\frac{1}{2}$. 3. $\frac{1}{2}$. 4. $\frac{1}{2}$. 5. 3. 6. 50.
 7. $\frac{1}{11}$. 8. $35\frac{1}{2}$. 9. .15. 10. $\frac{1}{2}$. 11. 25. 12. $\frac{1}{2}$. 13. $\frac{1}{2}$.
 14. 5. 15. $\frac{c}{d} = 3.1416$; $\frac{c}{r} = 6.2832$ 16. 3. 17. $\sqrt{2}$. 18. .7854.

Pages 171-174. — 1. 706.3 lb.; 437.5 lb. 2. 843.8 lb.; 375 lb.
 3. 556.3 lb.; 156.3 lb.; 556.3 lb. 4. .7 lb.; .4 lb.; .8 lb.
 5. 1.5 lb.; 3.9 lb. 6. 2.5 cu. ft. 7. 10.1 cu. ft. 8. 405 lb.; 3375 lb.
 9. 85.6 gal. 10. 6.6 lb.; 5.8 lb. 11. 258.75 lb. 12. 204.8 tons.
 13. 2.87 gm.; 0.507 gm.; 0.272 gm.; 0.091 gm. 14. 32; 15.86; 6; 14.29.
 15. No. 16. 0.97. 17. 3. 18. 1.58 oz. 19. 272 gm.
 20. 979.2 gm.; 13.8 lb. 21. .6 ft. 22. 7313 yd. 23. 4.875 lb.
 24. 226.64 lb. 25. 239.3 lb.; 3 in. 26. 8.8 in. 27. 539 lb.
 28. 2.8 lb. 29. 70170 lb. 30. 25422 lb.

Pages 176, 177. — 1. 100° ; $128\frac{1}{2}^\circ$; 135° ; $151\frac{1}{2}^\circ$; $182\frac{1}{2}^\circ$. 2. $83\frac{1}{2}^\circ$ F.;
 32.004° F. 3. $332\frac{3}{4}^\circ$; -39.439° ; 1292° ; 2012° ; 2552° . 4. 172.4° ; 95° ;
 674.6° ; 640.4° ; 2732° ; 320° . 5. 20.01074 ft. 6. 1.5004914 in.
 7. 1.2451825 in. 8. Because platinum and glass expand equally.
 Because copper and glass do not expand equally. 9. .111868 in.
 10. .12 ft. 11. .12432 in. 12. .0046 in. 13. 2.002484 in.;
 6.007432 in.; 12.00138 ft. 14. 9.99688 in. 15. .084 ft.
 16. 5.01746 cu. cm.; 5.0261 cu. cm.; 5.02048 cu. cm. 17. 428.009 cu. in.

Page 178. — 1. 10; $26\frac{1}{2}$; 1. 2. $31\frac{1}{2}$ cu. cm.; 12; $22\frac{1}{2}$ cu. cm.
 3. $1\frac{1}{2}$ sec.; $8\frac{1}{2}$ sec.; 170. 4. 4; 6. 5. (a) Yes; because $2:6=5:15$.
 (b) No; because $20:4$ is not equal to $5:2$. 6. (a) Yes; because
 $12:8=36:24$. (b) No; because $4:96$ is not equal to $7:164$.

Pages 181-184. — 1. 160 lb. 2. 4.91 in. 3. 100 lb. 4. 16 lb.
 5. 125 lb. 6. 96 lb. 7. $82\frac{1}{2}$ lb. 8. 12 lb. 9. 1508 lb.
 10. 12000 lb. 11. 42 lb. 12. $71\frac{1}{11}$ lb. 13. 3780 lb. 14. $42\frac{1}{2}$ lb.
 15. 2000 lb. 16. By $\frac{3}{8}$. 17. $666\frac{2}{3}$ lb. 18. (1) 75 lb.; (2) 90 lb.
 19. $177\frac{1}{12}$ lb. 20. $1085\frac{1}{2}$ lb. 21. 1.5 tons. 22. 80 lb. 23. 12 lb.
 24. 2639 lb. 25. 14.9 lb. 26. 25133 lb. 27. 35186 lb.
 28. 14 in. 29. (1) 90478 lb., approx.; (2) 226195 lb., approx.

Pages 185-189. — 1. 15 ft., 18 ft. 2. 217 sq. yd., approx.
 3. 75.54+ lb. 4. 524.2+ lb. 5. (1) 5000 ft., dist. from O;
 (2) 120 ft., dist. from O. 6. 68 ft. 7. (1) 50 ft.; (2) 108 ft.
 9. (1) 66 ft.; (2) 58.4 ft., approx. 10. 52488 cu. in. 11. (1) $16\frac{1}{2}$ in.;
 (2) $22\frac{1}{2}$ in. 12. 3.72 in. 13. .9 in. 14. $2\frac{1}{2}$ in. 15. $91\frac{1}{2}$ cu. cm.
 16. 6.73 in. 17. (1) 70 ft.; (2) $106\frac{2}{3}$ ft.; (3) 72 ft. 18. (1) 3.9 in.;
 (2) 12.5 in.; (3) 4.6 in.; (4) 12.8 in. 19. $5\frac{1}{2}$ in.

Pages 190, 191. — 1. (1) 64 ft.; (2) 144 ft.; (3) 256 ft.; (4) 400 ft.
 2. (1) 48 ft.; (2) 80 ft.; (3) 112 ft. 3. (1) 1600 ft.; (2) 304 ft.;
 (3) 57600 ft. 4. (1) 64 ft. per sec.; (2) 96 ft. per sec.;
 (3) 320 ft. per sec.; (4) 1920 ft. per sec. 5. 6 sec. 6. 3 sec.
 7. 100 ft. 8. 144 ft. 9. (1) 144 ft. per sec.; (2) 324 ft.
 10. 22 ft. 11. (1) 192 ft. per sec.; (2) 576 ft.

Pages 191, 192. — 1. $\frac{1}{2}$ sec., approx. 2. 4.3+ in. 3. 22 in. approx.
 4. 5.3+ sec. 5. .11 in. 6. Less than .01 in. 7. .11 in.

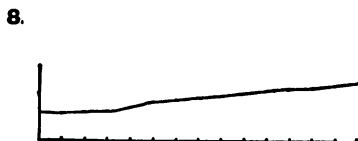
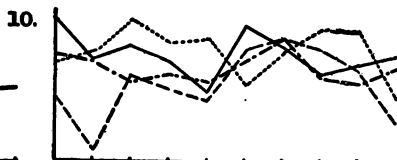
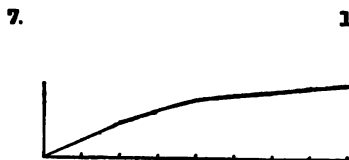
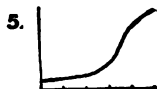
Pages 192, 193. — 1. $10\frac{1}{2}$ cu. yd. 2. $9\frac{1}{2}$ cu. in. 3. 1.361 gm.
 4. .5 qt. 5. 18 lb. 6. 150 lb.

Pages 193, 194. — 1. 4 candle power. 2. $10\frac{4}{5}$ candle power.
 3. 60 in. 4. 16 candle power. 5. 1.63+ candle power.
 6. 1200 candle power. 7. 52, approx.

Pages 195-197. — 1. (1) $\frac{1}{2}$ mm.; (2) 1 m. 2. 20 m. 3. 40 ohms.
 4. 1.5 mm. 5. 143.2 ohms. 6. (1) .5434 ohms; (2) 34.485 ohms.
 7. 382.8 ft. 8. 191.39 ft. 9. .3936 ohm. 10. .48 ohm.
 11. .912 ohm. 12. 51 in. 13. 300 volts. 14. 20 amperes.
 15. 60 volts. 16. .183+ amperes. 17. .3 ohm. 18. 1240 volts.

Pages 199-204.

1. North America _____
 South America _____
 Europe _____
 Asia _____
 Africa _____
 Australia _____
2. Mercury .
 Venus .
 Earth .
 Mars .
 Jupiter _____
 Saturn _____
 Uranus _____
 Neptune _____
3. America _____
 English _____
 German _____
 Italian _____
 Irish _____
 Polish _____
4. 1880 _____
 1890 _____
 1903 _____

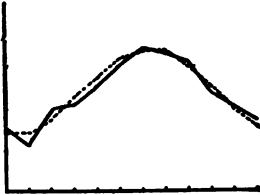


12

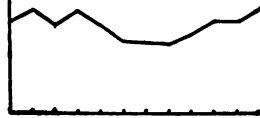
ANSWERS

[Pp. 199-204]

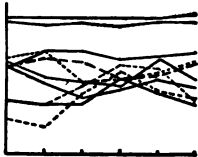
12.



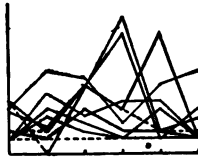
13.



14.



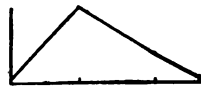
15.



16.



17.



18.



19.



20.



21.



PART IV.

- Pages 205-218. — 1. \$450; \$7500. 2. Wheat: $33\frac{1}{2}$ lb.; $13\frac{1}{2}$ lb.; $8\frac{1}{4}$ lb. Potatoes: $22\frac{1}{2}$ lb.; $7\frac{1}{2}$ lb.; 30 lb. 3. 7623 lb.; 3267 lb.; 8712 lb.; 261+.
4. \$4.05. 5. \$11 per acre. 6. 3240; 81 bu. 7. \$22.25.
8. 66 lb. peas to 175 lb. hay. 9. \$2.81. 10. \$1365.75; \$43.90.
11. Bal. \$732.95. 12. \$17,776. 13. 4 ft. 8 in. 14. Nearly 76 ft.
15. 23.27 cu. yd. 16. 1 ft. 9 in.; $317\frac{1}{2}$. 17. 4747.36 T.
18. About 30 in. 19. $58\frac{1}{2}$ gal. 20. 24,990 cu. ft. or nearly 926 cu. yd.
21. \$2.14. 22. \$48.80. (This answer is upon the assumption that a fraction of a barrel or of a load cannot be bought. Then 20 bbl. of cement and 16 loads of gravel are required.) 23. \$132.96. (This assumes that a fraction of a barrel cannot be bought.) 24. 5400 sq. in. 25. $\frac{1}{2}$; $\frac{3}{8}$; $\frac{1}{4}$; $\frac{1}{8}$. 26. \$111.60.
27. \$88.97. 28. \$347.57. 29. \$72. 30. 720. 31. \$47.30.
- (This considers that plaster can be bought in 100-lb. bags.) 32. \$145.15.
33. 7 bbl. 34. Yes. 7.41% was moisture. 35. 13.04%.
36. About 26 bd. ft. 37. 19 ft. 2.5 in. 38. Nearly 3150.
39. 169.65 T; 678.59 T. 40. 169.65 T. 41. 900; 1560; 4032. 42. 2 : 3.
43. About 7 sq. ft. 44. 38.5 T. 45. 2 ft. 2 in. 46. 21.7; 15.19; 2.17; 69.44; 104.16. 47. \$43.32 48. 1 ft. 7 in.; 1 ft. 1 in.
49. 1224 bu.; 73440 lb. 50. 11+ gal. 51. 589.05 sq. ft.
52. 18,095.62 lb. 53. 101,787.84 lb. 54. 28.06 lb. 55. 4 ft. 1 in.
56. 62,668.8 lb. 57. 5 ft. 5.3 in. 58. No. 59. 27,417.6 lb.
60. 6798 gal. 61. 10. 62. 559.55 lb. 63. About .527 of it.
64. About 707 to 272. 65. 8225.28 gal. 66. Nearly 4 ft. 9 in.
67. 1033.146 lb. 68. $4833\frac{1}{2}$ cu. yd. 69. 4687.5 lb. 70. 9,000,000 lb.
71. 5132.16 lb. 72. 157.464 lb. 73. 38+. 74. 3.386 ft.
75. 19.6 in. 76. 1.995 oz.; 24.75 oz. 77. 600. 78. 18-inch.
79. 7 lb.; 14 lb.; 154 lb. 80. $\frac{1}{2}$. 81. 12.5664 sq. in. 82. 25.39 lb.
83. 4.55 in. 84. 3713.96 sq. ft. 86. 76; 195.3. 87. 108+ lb.
88. 480. 89. 1.287 in. 90. .0029 in. 91. 2.08.

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